

Fiscal Monetary Services and Inflation*

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Abstract

In this paper I use a Fisher ideal index to track the monetary services provided by marketable US government debt. To do so, I first develop the theory necessary to consider using such a statistical index number, show how the value of these fiscal monetary services expand the fiscal capacity to borrow, and provide evidence that the monetary services are primarily safety services. I then use Jordà (2005) projections to estimate the impact of such monetary services on inflation. I find that a one-percent increase in fiscal monetary services produces a positive and statistically-significant inflationary response that peaks between four and five basis points and persists for eight months. Given that the average growth rate of the fiscal monetary services in my sample is 2.5 percent, the impact is also economically significant. Together, these results suggest that there is a monetary services channel to the fiscal theory of the price level.

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1 Introduction

The fiscal response to the COVID crisis in the US and the ensuing rise of inflation and interest rates has brought the concept of “fiscal capacity” back into the spotlight. Despite growing debt levels, borrowing costs trended downward from the 1980s until the US financial crisis in the last 2000s. Forecasters, despite accelerating debt levels, suggest that short-term interest rates will peak somewhere around two percent in the post-COVID cycle. All of this suggests that there must be more to the fiscal-capacity discussion than just the stock of outstanding principal values. If that additional component could be discovered, could it also shed light on theories like the fiscal theory of the price level (FTPL)?

Krishnamurthy and Vissing-Jorgensen (2012), Vissing-Jorgensen and Krishnamurthy (2013), and Nagel (2016)—to name just a few—have already established that the value of fiscal debt in the US is much more than the sum of its outstanding stock of principal. Caballero, Farhi and Gourinchas (2017) build on this idea to explain historically low borrowing costs in the face of historically high debt levels. More recently, Brunnermeier, Merkel and Sannikov (2020, 2022) have applied these concepts to the fiscal capacity and the FTPL literatures. They show that the demand side of this debt market raises the borrowing limits of the fiscal authority because the additional debt provides “transaction services,” which I’ll refer to more generally as monetary services.¹ Theoretically, they show that the inclusion of these monetary services—as well as a plausible bubble term—disrupt the traditional FTPL channel (see Leeper, 1991, as a seminal example), but introduce the possibility of other channels. While the measurement of a possible bubble term is likely impossible, uncovering the extent of these fiscally-provided monetary services would be significant contribution to our understanding of debt dynamics and their impact on the economy as a whole.

The purpose and contribution of this paper is two-fold. First, I measure the monetary services established as theoretically important in the literature. This measure provides additional insight not only into how debt and its dynamics impact the greater economy, but also how global events impact the monetary services of outstanding fiscal debt. If fiscal monetary services do provide additional fiscal capacity and influence inflation dynamics as Brunnermeier *et al.* (2020, 2022) theorize, then understanding the extent of the impact is vital to policy makers. Second, I provide evidence of a monetary services channel to the fiscal theory of the price level. The debate between the monetarist view of inflation and those behind the FTPL has been long and arduous. Showing that fiscal debt influences inflation through its monetary properties suggests that these two theories may not be as different as suspected and provides a potential bridge between the two. Together, the results within this paper provide a significant contribution to the FTPL, monetary aggregation, fiscal capacity, and inflation literatures.

¹ These monetary services generally include the aforementioned transactions services as well as other attributes such as liquidity, safety, etc.

The first result of this paper is the derivation of the underlying theory for the quantity index of interest. As is noted in the extensive monetary aggregation literature (e.g. Barnett, 1978, 1980; Barnett and Serletis, 2000), a statistical index used to track a true aggregate like the one considered here must be derived from an optimizing agent. Therefore, I consider a partial equilibrium model of the representative household which incorporates the monetary services of short- and long-term fiscal debt. The resulting holding-period user costs of these securities include both the respective coupon payments as well as the expected future capital gains. I then re-derive the budget constraint as done by Brunnermeier *et al.* (2020) to show that the value of these fiscal monetary services (the quantity index multiplied by its price dual) wholly adds to the fiscal capacity of the government. Having shown the potential importance of these fiscal monetary services, the natural next step is to assess how this quantity index has evolved over time.

In evaluating the index and comparing it to the ubiquitous simple sum aggregate, I then derive the growth rate of the monetary services provided by the fiscal authority. Isolating the growth of these fiscal monetary services is calculated as the growth rate of the Fisher ideal index—which incorporates both the monetary services and quantities—less that of the simple sum aggregate—which is a pure quantity measurement. Fluctuations in this growth rate, and the historical events surrounding them, suggest that I am indeed capturing what is intended. For example, I find a sharp and sustained rise in the growth of fiscal monetary services throughout the period encompassing the American Recovery and Reinvestment Act of 2009 and the European debt crisis that followed shortly thereafter. I also find a sharp contraction in these fiscal monetary services during the “dash for cash” liquidity squeeze seen in the early months of the 2020 pandemic. Having this measure of fiscal monetary services provides a new data point in evaluating the impact of fiscal deficits and debt on the economy at large.

Why kind of monetary services do Treasury securities provide? Generally, they are considered to be both safe and liquid. I test these ideas empirically, finding that Treasury securities do reduce the price of safety in the market, but that they—at best—have no impact on liquidity and may even reduce liquidity in the market. While Treasury bills are considered to be nearly as liquid as any other asset out there, the full portfolio of Treasury debt doesn’t necessarily share that attribute. And since each issuance of new Treasury debt extracts reserves and currency from the market, the net impact is likely negative. Thus, while we consider Treasury debt to be safe and liquid overall, it seems that the monetary services provided center around safety.

Lastly, I show that an increase in fiscal monetary services has a positive, persistent, and statistically significant impact on the inflation rate. Perhaps the most important result in this paper, a one-percent increase generates an elevated inflation rate that peaks between four and five basis points and lasts for eight months. Put another way, a one-time increase in fiscal monetary services causes a permanent increase in the price level. This result is considered economically significant since, while the shock of interest is only one percentage point, the average growth rate in the sample is approximately 2.5 percent and frequently rises into the 5-10 percent range. Thus, sudden changes in the

monetary services provided by the fiscal authority have the potential to dramatically influence the price level dynamics. This provides a new piece of evidence for the fiscal theory of the price level (FTPL), though the pricing dynamics are derived not from the quantity of debt in existence, but rather in the monetary services it provides. To put it simply, “inflation is always and everywhere a monetary phenomenon,” even when it’s fiscal.

The remainder of this paper is organized as follows. Section 2 presents the underlying index number theory and considers some of the hurdles with applying it to the available data. Section 3 develops the theory to motivate a proper statistical index number. Section 4 derives the Fisher ideal index that tracks the true aggregate of marketable fiscal debt. Section 5 presents the Fisher ideal index and compares it to the ubiquitous simple sum aggregate. Section 6 then tests the ability of this metric to forecast inflationary dynamics. Section 7 concludes.

2 Measuring the Monetary Services of Government Debt

Measuring the monetary services of fiscal debt specifically is a novel concept, but the idea of considering the monetary services of financial assets in general is not new. In this section I briefly touch on the monetary aggregation literature, motivate the need to track such an aggregate, and consider the hurdles in applying this established theory to a new literature.

2.1 Monetary Aggregation as a Guide

Fiscal debt is ubiquitously measured as the sum of outstanding principal values. To assess the monetary services, one needs to aggregate fiscal debt as it is done in the monetary aggregation literature. There the preferred method is typically a Törnqvist-Theil Divisia index

$$\log M_t^d - \log M_{t-1}^d = \sum_{i=1}^N s_{i,t}^* (\log m_{i,t} - \log m_{i,t-1}),$$

where M_t^d is the level of the Divisia monetary aggregate, $m_{i,t}$ is the nominal value of asset i , and $s_{i,t}^*$ is the average value share (or weight) on the marginal change in asset i between $t - 1$ and t . The key attribute of this measure lies within the value share itself, where

$$s_{j,t} = \frac{\eta_{j,t} m_{j,t}}{\boldsymbol{\eta}_t \mathbf{m}'_t}$$

corresponds to the weight on the marginal change in financial asset j . Here, \mathbf{m}_t and $\boldsymbol{\eta}_t$ are $N \times 1$ vectors of nominal quantities and user costs, respectively. Notice that there are two mechanisms at work here. First, a lower interest rate relative to the benchmark

generates a higher user cost, which can put upward pressure on the weight. This reflects the money-ness of a financial asset and is a direct function of its demand. Second, the nominal values are also incorporated into the share value, so that even if a particular asset is very money-like, it still will not add much to the aggregate at smaller nominal amounts. Thus, it is important to note that an increase in an assets user cost does not necessarily imply an increase in its share value.

2.2 Market Segmentation as Application Motivation

The general reasoning behind the use of theoretically-true aggregates is that the simple-sum derivation implies that the underlying assets are perfect substitutes. An easy way to assess the substitutability is to check for a yield-to-maturity (YTM) spread between Treasury notes and Treasury bonds of the same remaining time to maturity. That is, comparing the YTM of a Treasury note that matures in t periods with a Treasury bond that also matures in t periods. Technically speaking, the only difference between these securities on the secondary market is the label attached to their initial maturity. So we should expect that a note and bond would be perfect substitutes and thus carry the same YTM. Figure 1 effectively presents their median yield curves from 1970-2020. This shows that, while not constant, there is a persistent spread between the yields on these two instruments.² Perhaps more surprising is that bonds sell at a premium relative to notes of the same time to maturity. But even this is not constant, as the median overall spread has fluctuated over time as shown in Figure 2. A likely explanation of this situation is a simple extension of the work of Amihud and Mendelson (1991): relative liquidity. Figure 3 presents a comparison of the bid-ask ratios, suggesting that the Treasury bond market is more liquid than that of the Treasury note beyond the two-year mark. These snapshots show that, even when identical on paper, the UST market is quite segmented, and the assets therein cannot be perfect substitutes.³

2.3 Hurdles in the Application

One drawback of the Törnqvist-Theil Divisia index is that it does not handle assets coming into and out of the market ($x_{j,t} = 0$ for some t) very well. When calculating general monetary aggregates, this problem does not arise very often and is managed by imputing a reservation price and switching to a Fisher ideal index for those time periods. Given the frequent changes in Treasury policy and the categorization I use, zero values are commonplace in this analysis.

² The yields are calculated by first grouping securities into those denoted as *Notes* and *Bonds*, then by the number of quarters remaining to maturity. That is, a security set to mature within three months is considered a security that will mature within one quarter, etc. These groupings are done for every month between January 1970, and December 2020. The yields shown are the median values of each across the time period. More information about this categorization method can be found later.

³ This also lends credence to the use of “preferred habitat” assumptions in structural models to motivate the the term structure of the interest rates.

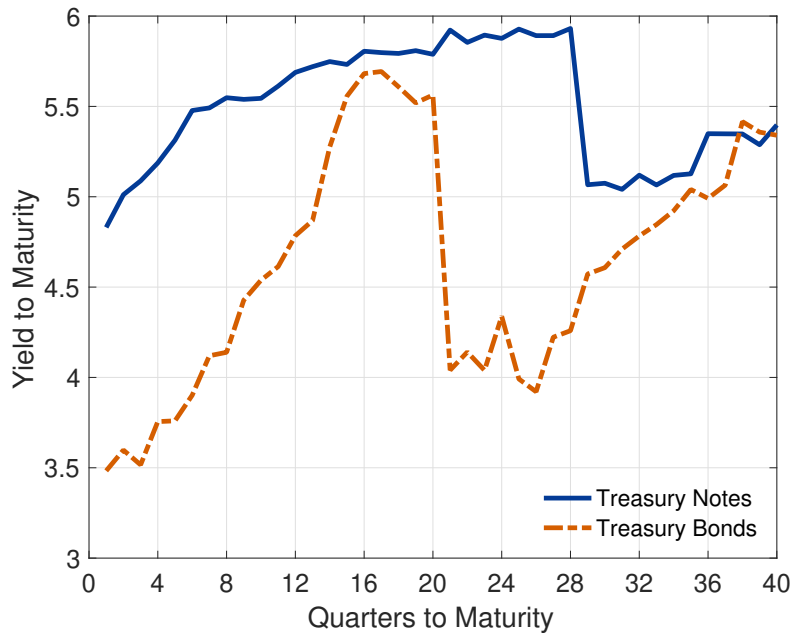


Figure 1: Median Treasury Bond and Note Yields with Same Time to Maturity (1970-2020)

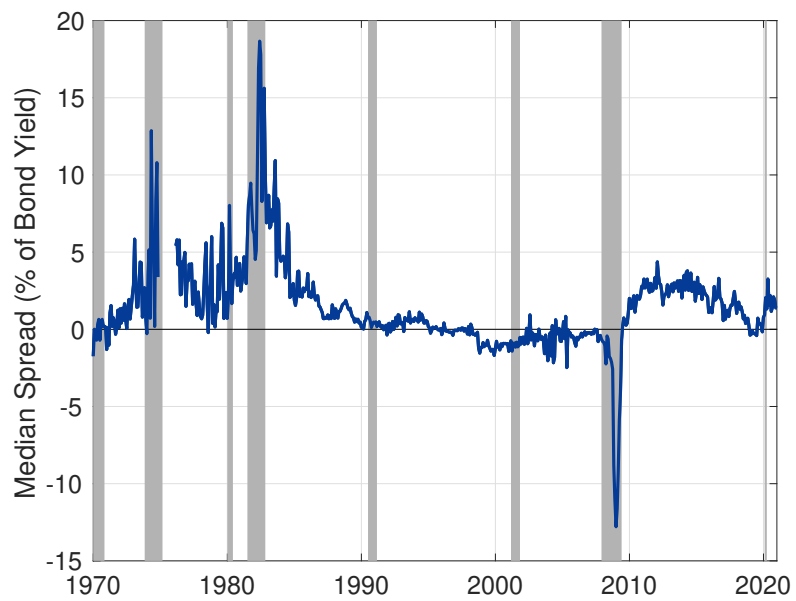


Figure 2: Median Treasury Note–Bond Yield Spread with Same Time to Maturity (1970-2020)

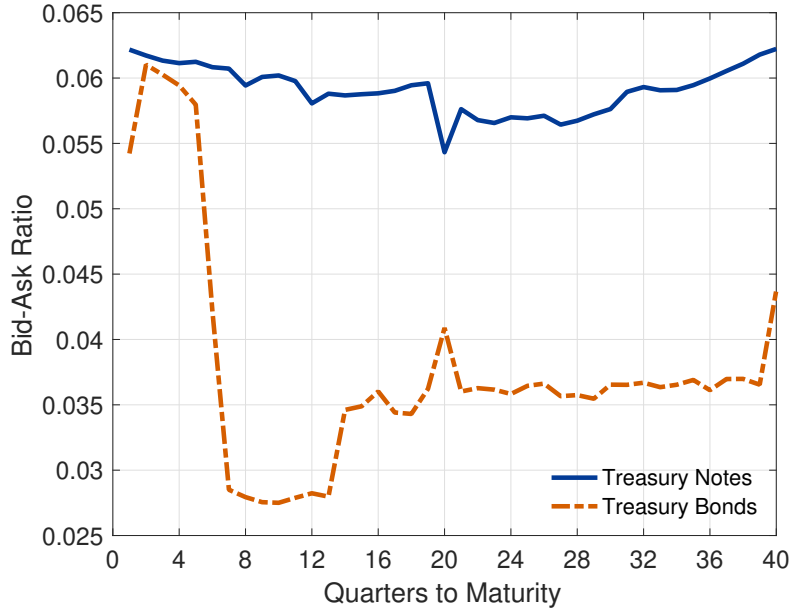


Figure 3: Median Treasury Bond and Note Bid-Ask Ratios with Same Time to Maturity (1970-2020)

To better account for zero values, I utilize the Fisher ideal index for the full sample. This index still utilizes the user cost approach discussed above and is shown to be a Diewert-superlative index number like the Divisia index (Diewert, 1976); but happens to be more applicable to the constraints of this particular dataset. The Fisher ideal index (in levels) is expressed as

$$M_t^f = M_{t-1}^f \left[\frac{(\eta_t \mathbf{m}'_t)(\eta_{t-1} \mathbf{m}'_t)}{(\eta_t \mathbf{m}'_{t-1})(\eta_{t-1} \mathbf{m}'_{t-1})} \right]^{\frac{1}{2}}, \quad (1)$$

where M_t^f denotes the new aggregate and the rest of the notation is defined above. Since there is no initial value of this index M_0^f , the aggregate is first calculated in growth rates and then converted to a quantity index.

3 Theory

As Barnett (1980) outlines, aggregation theory relies on known, exact functional forms with estimable parameters. The functions of interest are typically utility and production functions, which are often impossible to know. This is why I rely on statistical index numbers, whose theory only relies on the existence of maximizing behavior. From this optimizing behavior we can derive the user costs of the underlying components and bypass the unknown parameters because the resulting index numbers are not dependent

on any specialized properties of the aggregator function. That is, while I'm not deriving the true aggregate itself, the resulting quantity index will track the true aggregate. A complete guide to index number theory and its application to monetary aggregation can be found in Barnett and Serletis (2000).

A proper quantity index requires a price that is derived from an optimizing agent. Thus, motivating the measurement of marketable government debt requires a partial equilibrium model that derives both the period-by-period user cost of holding government debt as well as the budgetary constraints the fiscal authority encounters. This model incorporates both short- and long-term debt issued by the government, as well as an alternative long-term asset that will act as the benchmark asset. In this model, I refer to the alternative asset as “capital,” though it could also be motivated in some other fashion.

3.1 Long-Term Asset Dynamics

The long-term government bonds and capital both evolve in a similar fashion. As described by Krause and Moyen (2016), each period new nominal long-term government bonds $B_t^{L,n}$ are issued, which are added to the stock of outstanding long-term debt B_t^L . A portion $\alpha \in (0, 1)$ of the previous period's stock of long-term bonds mature, while the remaining $(1 - \alpha)$ remain in the stock of outstanding long-term debt. The maturity of these bonds is therefore $1/\alpha$. Together, the stock of outstanding long-term debt evolves such that

$$B_t^L = (1 - \alpha)B_{t-1}^L + B_t^{L,n}. \quad (2)$$

The average nominal interest rate paid on the stock of outstanding long-term debt r_t^L evolves such that

$$r_t^L B_t^L = (1 - \alpha)r_{t-1}^L B_{t-1}^L + r_t^{L,n} B_t^{L,n}, \quad (3)$$

where $r_t^{L,n}$ is the interest rate on newly-issued long-term bonds.

Capital is structured in a similar fashion, with the total stock of outstanding capital evolving according to

$$K_t = (1 - \delta)K_{t-1} + I_t, \quad (4)$$

where I_t is new investment in capital and $\delta \in (0, 1)$ designates the portion of the outstanding stock of capital that matures each period. This implies that the maturity of capital is $1/\delta$. The average interest rate paid on the capital stock outstanding R_t is derived by

$$R_t K_t = (1 - \delta)R_{t-1} K_{t-1} + R_t^n I_t, \quad (5)$$

where R_t^n is the interest rate on newly-issued capital.

3.2 Representative Household

The representative household in this model works l_t hours each period for nominal wage W_t , less an income tax rate τ_t . It also earns income through its previous investments in capital $(\delta + R_{t-1})K_{t-1}$, short-term (one-period) bonds $(1 + r_t)B_{t-1}$, long-term bonds $(\alpha + r_{t-1}^L)B_{t-1}^L$, and profits from a continuum of intermediate goods-producing firms $\int_0^1 \Pi_t(s)ds$.⁴ This income is spread between a real consumption good c_t at price p_t as well as new investments in nominal short-term bonds B_t , long-term bonds $B_t^{L,n}$, and capital I_t . Combined, the household's period-by-period budget constraint can be expressed as

$$B_t + B_t^{L,n} + p_t c_t + I_t = (\delta + R_{t-1})K_{t-1} + (1 + r_{t-1})B_{t-1} + (\alpha + r_{t-1}^L)B_{t-1}^L + (1 - \tau_t)W_t l_t + \int_0^1 \Pi_t(s)ds. \quad (6)$$

The household objective is to maximize utility over the real consumption good, the real monetary services provided by its portfolio of government bonds, and leisure

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + v\left(\frac{M_t}{P_t}\right) + x(1 - l_t) \right\}, \quad (7)$$

where $u(\cdot)$, $v(\cdot)$, and $x(\cdot)$ are increasing, concave functions. The monetary services provided by the portfolio are captured by a constant elasticity of substitution function

$$M_t = \left[\lambda^{\frac{1}{\sigma}} B_t^{\frac{\sigma-1}{\sigma}} + (1 - \lambda)^{\frac{1}{\sigma}} B_t^{L,n \frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}, \quad (8)$$

where $\lambda \in [0, 1]$ dictates the weight of each government debt security and σ dictates the elasticity of substitution between the two assets.

Solving the household's problem results in the following dynamic conditions:

$$1 - \gamma_{2,t} \left(\frac{\lambda M_t}{B_t} \right)^{\frac{1}{\sigma}} = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1 + r_t}{\pi_{t+1}} \right], \quad (9)$$

$$\begin{aligned} 1 - \gamma_{2,t} \left(\frac{(1 - \lambda)M_t}{B_t^L} \right)^{\frac{1}{\sigma}} + \gamma_{4,t} (r_t^L - r_t^{L,n}) \\ = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}} \left\{ 1 + r_t^L + (1 - \alpha)\gamma_{4,t+1} (r_t^L - r_{t+1}^{L,n}) \right\} \right], \quad (10) \end{aligned}$$

⁴ The δ considered here can deviate from that in (4) if depreciation is considered. In this present study, however, we assume zero depreciation and treat "capital" as an alternative long-term security similar to the long-term government bond. Additionally, the inclusion/exclusion of the corporate profit term does not alter the results of this partial equilibrium setup, but would be needed in a full, general equilibrium presentation.

and

$$1 + \gamma_{3,t}(R_t - R_t^n) = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}} \{1 + R_t + (1 - \delta)\gamma_{3,t+1}(R_t - R_{t+1}^n)\} \right], \quad (11)$$

The solution technique to the household's problem can be found in Appendix A. Here, $\mu_{1,t} = u'(c_t)$, $\gamma_{2,t} = v'(\cdot)/u'(\cdot)$, and $\gamma_{3,t}$ and $\gamma_{4,t}$ are the prices of the long-term assets B_t^L and K_t , respectively. These prices evolve according to

$$\gamma_{3,t} = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}} \{1 + (1 - \delta)\gamma_{3,t+1}\} \right], \quad (12)$$

and

$$\gamma_{4,t} = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}} \{1 + (1 - \alpha)\gamma_{4,t+1}\} \right]. \quad (13)$$

The user costs of the short- and long-term bonds are

$$\eta_t = \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n - (1 - \delta)\gamma_{3,t+1}\Delta R_{t+1}^n - r_t \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n - (1 - \delta)\gamma_{3,t+1}\Delta R_{t+1}^n \right\} \right]}, \quad (14)$$

and

$$\eta_t^L = \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n - (1 - \delta)\gamma_{3,t+1}\Delta R_{t+1}^n - r_t^{L,n} + (1 - \alpha)\gamma_{4,t+1}\Delta r_{t+1}^{L,n} \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n - (1 - \delta)\gamma_{3,t+1}\Delta R_{t+1}^n \right\} \right]}, \quad (15)$$

respectively. The derivations of these user costs can be found in Appendix B.

3.3 Fiscal Capacity Considerations

Recently, Brunnermeier, Merkel and Sannikov (2020) and Brunnermeier *et al.* (2022) have put forth theories suggesting that government debt constraints need to be augmented for both the service flows (transaction/monetary services) and a bubble term. Here I augment this idea with the specifics of the model above, providing a deeper look at how we can ascertain those services flows in particular. Consider the simple budget constraint of the government in this model

$$(1 + r_{t-1}) \frac{B_{t-1}}{p_t} + (\alpha + r_{t-1}^L) \frac{B_{t-1}^L}{p_t} = \frac{B_t}{p_t} + \frac{B_t^{L,n}}{p_t} + s_t, \quad (16)$$

where s_t is denotes the real primary surplus. Now incorporate (2) into the above equation to get

$$(1 + r_{t-1}) \frac{B_{t-1}}{p_t} + (1 + r_{t-1}^L) \frac{B_{t-1}^L}{p_t} = \frac{B_t}{p_t} + \frac{B_t^L}{p_t} + s_t. \quad (17)$$

Since (9) and (10) must hold,

$$\begin{aligned}
\frac{B_{t-1} + B_{t-1}^L}{p_t}(1 + r_{t-1}) &= s_t - (r_{t-1}^L - r_{t-1})\frac{B_{t-1}^L}{p_t} + \beta\mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t + B_t^L}{p_{t+1}}(1 + r_t) \right] \\
&\quad - \beta\mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t^L}{p_{t+1}}(1 + r_t) \right] + \beta\mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t^L}{p_{t+1}}(1 + r_t^L) \right] \\
&\quad + \gamma_{2,t} \left(\frac{\lambda M_t}{B_t} \right)^{\frac{1}{\sigma}} \frac{B_t}{p_t} + \gamma_{2,t} \left(\frac{(1-\lambda)M_t}{B_t^L} \right)^{\frac{1}{\sigma}} \frac{B_t^L}{p_t} \\
&\quad + \beta\mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t^L}{p_{t+1}} \left\{ 1 + (1-\alpha)\gamma_{4,t+1} (r_t^L - r_{t+1}^{L,n}) \right\} \right] - \gamma_{4,t} (r_t^L - r_t^{L,n}) \frac{B_t^L}{p_t} \quad (18)
\end{aligned}$$

Including (13) and some rearranging, we can simplify the above equation to

$$\begin{aligned}
\frac{B_{t-1} + B_{t-1}^L}{p_t}(1 + r_{t-1}) &= s_t - (r_{t-1}^L - r_{t-1})\frac{B_{t-1}^L}{p_t} + \beta\mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t + B_t^L}{p_{t+1}}(1 + r_t) \right] \\
&\quad + \beta\mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t^L}{p_{t+1}}(1 + r_t^L - (1-\alpha)\gamma_{4,t+1}\Delta r_{t+1}^{L,n} - r_t) \right] \\
&\quad + \gamma_{2,t} \left(\frac{\lambda M_t}{B_t} \right)^{\frac{1}{\sigma}} \frac{B_t}{p_t} + \gamma_{2,t} \left(\frac{(1-\lambda)M_t}{B_t^L} \right)^{\frac{1}{\sigma}} \frac{B_t^L}{p_t}. \quad (19)
\end{aligned}$$

The last two terms are of particular importance here. Incorporating the definition of the monetary services aggregate reduces the above expression to

$$\begin{aligned}
\frac{B_{t-1} + B_{t-1}^L}{p_t}(1 + r_{t-1}) &= s_t - (r_{t-1}^L - r_{t-1})\frac{B_{t-1}^L}{p_t} + \beta\mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t + B_t^L}{p_{t+1}}(1 + r_t) \right] \\
&\quad + \beta\mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t^L}{p_{t+1}}(1 + r_t^L - (1-\alpha)\gamma_{4,t+1}\Delta r_{t+1}^{L,n} - r_t) \right] + \gamma_{2,t} \frac{M_t}{p_t}, \quad (20)
\end{aligned}$$

where it can be shown that

$$\gamma_{2,t} = \left[\lambda \eta_t^{\sigma-1} + (1-\lambda) (\eta_t^L)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \quad (21)$$

is the price dual for the monetary services aggregate.

There are a couple of items to point out with regards to the above expression. First, fiscal capacity intuitively diminishes as long-term rates rise relative to short-term rates. This term is reflecting the convenience yields that the government enjoys on its short-term debt. Second, the fourth term on the right side suggests long-term debt can improve fiscal capacity via its longer maturity and a higher expected one-period return over the short-term debt. That is, while capacity shrinks with higher long-term interest rates, demand for the debt increases with higher expected holding-period returns. Lastly, fiscal capacity increases with the value of the monetary/transaction services the debt provides. Thus, the monetary services provided by government debt issuances is directly captured in the government's budget constraint in a similar fashion to that shown in Brunnermeier *et al.* (2020).

4 Data and Methodology

In this section, I construct the Fisher ideal quantity index described in Section 3 from data compiled by the Center for Research in Securities Prices on marketable Treasury securities. Non-marketable debt is typically held by intergovernmental agencies, and is therefore not typically considered to be a significant burden.⁵ The Treasury began its “regular and predictable” debt issuance campaign in the 1970s and data regarding the spot and forward interest rates goes back to January 1977. Therefore the variables constructed here will cover the February 1977 to December 2020 period.

The first issue that needs to be addressed is essentially analogous to new-product bias. As (1) shows, the calculation of the growth rates requires both current and lagged quantities and user costs. If I were to treat each issuance as a separate $m_{i,t}$, there would be distortions at issuance and maturity months. For example, in the issuance month of $m_{i,t}$, the preceding month’s yield $r_{i,t-1}$ of that particular issuance doesn’t exist, which means its user cost $\eta_{i,t-1}$ also doesn’t exist. Therefore, an assumption would need to be made regarding this lagged user cost. Feenstra (1994) and others have outlined a theoretically justified solution to this problem in which the price is set at its reservation level, where the quantity demanded would equal zero in the preceding month.⁶ It’s also well documented that on-the-run bonds sell at a premium over their off-the-run counterparts, suggesting $\eta_{i,t-1} > \eta_{i,t}$, but by how much? Estimating the reservation price of each issuance in each month over the sample period would seem to be too technically burdensome. With how distortionary this bias can be, some adjustments and/or assumptions are needed.

To reduce the magnitude of this issue, I cluster the assets into groups that are most likely to be perfect substitutes in any given month. While there may be discrepancies in on-the-run/off-the-run securities, they should average out within the clusters. Based on the work of Amihud and Mendelson (1991) as well as the descriptive statistics in Figures 1 and 3, the securities need to be separated across their designation of *bills*, *notes*, and *bonds*. Since 1997, the Treasury has issued Treasury Inflation-Protected Securities (TIPS), which yield a real rate of interest instead of the traditional nominal yield. These have been shown to be less liquid than their nominal counterparts, so I create two additional categories of *TIPS notes* and *TIPS bonds*. Lastly, as these securities mature, their yields tend to fall with the trend of the constant maturity yield curve, reflecting the changing monetary services they provide. Therefore, each of these five categories needs to be further segmented by their time to maturity. Since bonds

⁵ The inclusion of only publicly-held, marketable US Treasury debt would be the optimal choice here, but data restrictions keep me from assessing this. For instance, the CRSP data doesn’t include the amount of each T-bill issuance held publicly. Another option could be to exclude the issuances held by the Federal Reserve, since that is the primary non-public holder of US Treasuries, but the SOMA data only goes back to 2003. Future research could apply this methodology to the shortened timeframe.

⁶ The Center for Financial Stability, which publishes data on Divisia monetary aggregates, delays the inclusion of new monetary assets for a few months. In this situation, however, the regular issuance and maturity of short-term debt make this strategy problematic.

and longer notes are issued on a quarterly basis, it's common to have individual months where there are no securities of a certain type.⁷ A fine-grained segmentation such as this would again be subject to a large amount of new-product bias, so I define each sub-category as the quarters-to-maturity. That is, for securities that will mature over the next one-to-three months, they are categorized as maturing within one quarter. A thirty-year threshold on bonds suggests 120 quarters-worth of categories, but there were a series of longer-term bonds that were issued between 1953 and 1965, which could impact the first years of the analysis. Therefore, the number of quarters-to-maturity categories is set at 160, or forty years. Overall, when accounting for both the types of securities and the quarters to maturity, there are 800 categories.⁸

The next step in this process is to identify the proper interest rates from the theory above. Since the user costs represent the holding period return and incorporate the interest rates paid on newly-issued securities (r_t and $r_t^{L,n}$) and not on the average interest rate paid out on outstanding debt (r_t^L), the most accurate interest rates to consider are the coupon rates. Additionally, the term $(1 - \alpha)\gamma_{4,t+1}\Delta r_{t+1}^{L,n}$ represents the expected capital gains of the long-term bond. To account for these expectations, I use the current price of the bonds adjusted for its maturity, multiplied by the difference between the one-month-ahead forward rate and spot rate for each bond's particular month and maturity. For TIPS, I consider the real spot and forward yield curves in the calculation of expected capital gains. While forward rates incorporate more than just future expectations of current spot rates, the impact of any term premium looking only one month ahead should be negligible.

The rate considered for each basket of securities is the quantity-weighted average over the issuances therein, though a further assumption is needed to address missing values/new-product bias. As discussed above, even the broader categories used here do not ensure that there are no issues with new-product bias. Since most of the empty categories in this situation are simply a cluster of maturing debt through time, and not new issuances of debt, I use the linear interpolation of the rates across the maturities instead of calculating a reservation rate. This ensures that I'm capturing the true shape of the yield curves used.

The benchmark rate used here is the maximum holding-period return of the baskets of securities considered each month, plus twenty-five basis points.⁹ This is similar to the

⁷ These notes and bonds are typically subject to reopenings in the months following the initial issuance, but the maturity dates are unchanged. For example, if a 30-year bond is issued in February, a reopening of that issuance may be offered in March, but the time-to-maturity on that reopening is 29 years, 11 months.

⁸ Note that, based on (1), baskets that are consistently empty do not impact the measurement, allowing me to keep it consistent across the types of securities. So while there are 800 total baskets, a large number of them will not contribute to the measurement, but it does provide the flexibility needed for the aggregate over the almost fifty-year analysis.

⁹ Previous iterations of this measurement attempted to use the Baa Corporate Bond yield, which would have accounted for both the liquidity and safety attributes of Treasury securities. However, even the addition of 200 basis points did not ensure that all user costs were positive across the sample

strategy used by the Federal Reserve in their calculation of Divisia monetary aggregates and is common in other derivations of user costs.

5 Results

5.1 The Measure

The month-over-month growth rates of both the true aggregate and its simple sum counterpart are constructed from the data. This indirect approach to the aggregates' levels is necessary as there is no initial value from which to begin the calculation of (1). Simply taking the logarithm of (1) yields the growth-rate variation used. The general levels of the growth rates are similar, so I present their spread in Figure 4. To aid in the visual analysis, the 12-month moving average (centered on the seventh month) spread of these growth rates is superimposed. As can be seen, the true aggregate tends to grow faster than the simple sum aggregate, but there are also periods of relatively slow growth.

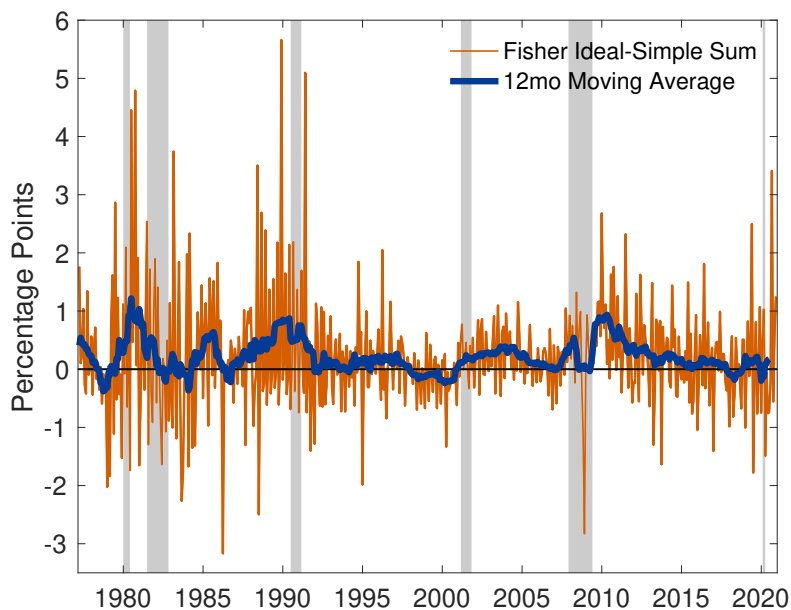


Figure 4: Month-over-month Growth Rate Spread

Since the true aggregate incorporates both the quantities of the underlying assets and the monetary services they provide, the spread intuitively reveals the growth of these

period, especially in the volatile years of 1979–1981 and long periods of time between 2014 and 2016. Adjustments to create that kind of spread for the early years also had the drawback of washing out the differences in user costs in the later parts of the sample period.

monetary services alone. That is, I am essentially factoring-out the growth that comes purely from the issued principal values. The spread in Figure 4, therefore, represents the growth of fiscally-provided monetary services. This figure shows us that the Treasury generally adds to the stock of monetary services over time, with notable exceptions in the late-1990s when the government was running a fiscal surplus. In that instance, the growth rates suggest that the monetary services provided were shrinking along with the principal values.

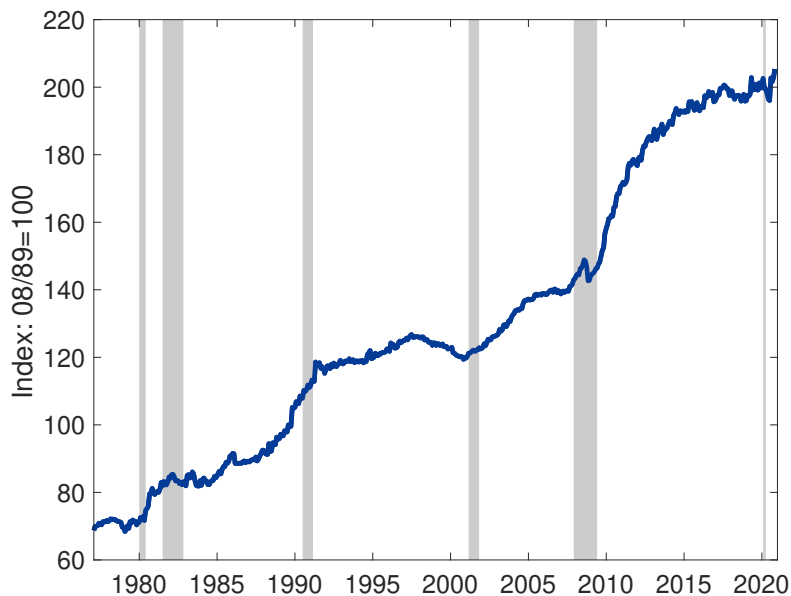


Figure 5: Monetary Services Level

Using these growth rate spreads, I derive an index that tracks the stock of monetary services, shown in Figure 5. Multiple theories have claimed that Treasury securities provide services above and beyond their principal value, but this is the first attempt to measure such services. The base month considered here is August 1989 due to the relative flatness of the yield curve at that time. When the yield curve is flat, the differences in the user costs are minimal, suggesting that the underlying assets are perceived as near-perfect substitutes and collapsing the Fisher ideal index into a simple sum aggregate.

5.2 What Services do Treasury Securities Provide?

The motivation and theory developed above claims that the Treasury provides monetary services above and beyond the sheer quantity of outstanding principal values. But what services are they actually providing?

To assess which services are provided by the Treasury, I use Jordà (2005) projections to estimate the impact of an increase in these measured monetary services on the price

of safety and liquidity. This is in a similar vein to the work of Krishnamurthy and Vissing-Jorgensen (2012) and others to study the impact of increased debt levels on the financial market attributes.

Exogenous shocks to the level of monetary services m_t shown in Figure 5 are identified using a simple OLS forecasting approach

$$m_t = c + \Gamma(L)X_t + \epsilon_t, \quad (22)$$

where c is a constant term and X_t includes the Wilshire 5000 stock price index, the trade-weighted US dollar index, the index of consumer sentiment from the University of Michigan's Surveys of Consumers, the simple-sum aggregate of outstanding principal values, and the dependent variable. These are chosen because much of the monetary services provided by these fiscal securities is determined by the economic environment and the expectations thereof. The residuals ϵ_t are the estimated exogenous shocks to fiscal monetary services. Four lags of the independent variables are considered based on AIC.

Impulse response functions are estimated via the linear function

$$z_{t+h} = c_t + \Phi(L_h)y_t + \beta_h\epsilon_t + \varepsilon_{t+h}, \quad (23)$$

where h is the forecast horizon, z_{t+h} is the dependent variable, ϵ_t is the identified shock of choice, y_t is the vector of control variables including lags of up to L_h , and β_h form the estimated impulse response functions. The variable c_t is a vector of trends includes the constant term as well as time trends ranging from linear to fourth-order.

For this exercise, there are two dependent variables of interest: the spread between the Baa and Aaa corporate bond yields, which proxies as a safety premium, and the spread between the Aaa corporate bond yield and the 10-year Treasury constant maturity rate, which proxies as a liquidity premium. The independent variables y_t in the model include the derived measurement, the simple-sum aggregate of outstanding principal values, the 10-year/3-month yield curve spread, the equity market volatility tracker, and dummy variables representing recessions and the zero-lower bound. A horizon of eighteen months is considered, lags are chosen at each horizon by AIC with maximum number of lags set at twelve, and Newy-West standard errors are used.

The estimated response functions and their 68- and 95-percent confidence intervals are presented in Figures 6 and 7. The Baa–Aaa spread (Figure 6) increases on average in response to a shock to fiscal monetary services, becoming statistically significant at the eight-month mark and remaining elevated for approximately five months. That is, it causes the price of safety to fall. This would correspond to an increase in the supply of safe assets in the market, suggesting that safety is, indeed, one of the monetary services provided. The impact on liquidity, however, is counter-intuitive given the standard narrative. Figure 7 suggests that an increase in these monetary services has little impact on market liquidity and even leads to an increase in the price of liquidity at horizons in excess of one year.

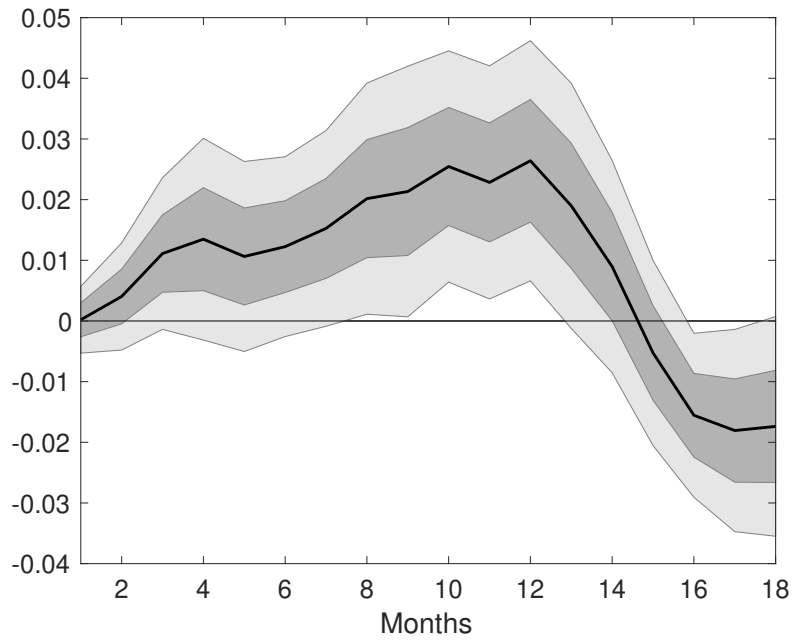


Figure 6: Impact of Monetary Services on Baa-Aaa Spread

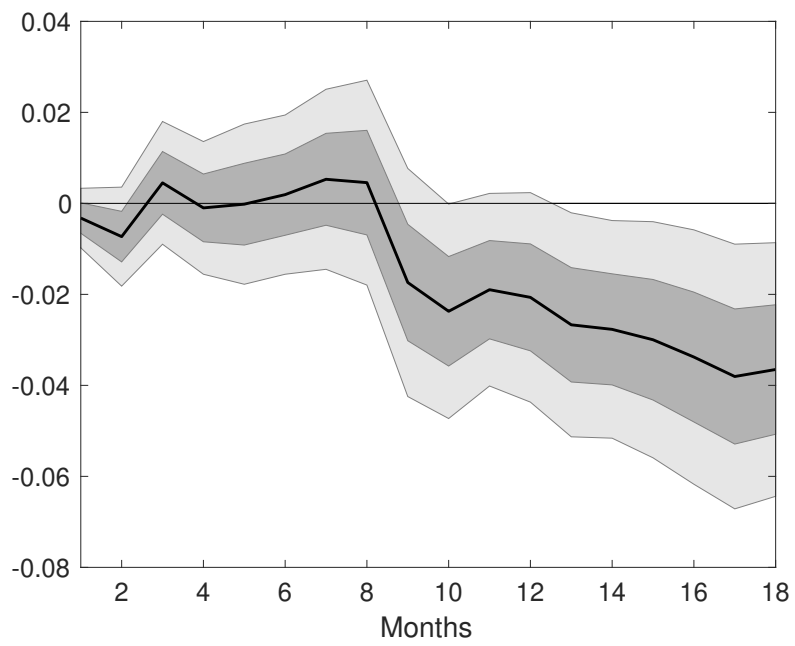


Figure 7: Impact of Monetary Services on Aaa-10yr Spread

One potential explanation of the lack of liquidity provided by the Treasury corresponds to the line of reasoning by Singh and Stella (2021) regarding quantitative easing. That is, issuing new bonds requires the extraction of more liquid forms of money from the economy. So, on net, and despite US Treasuries’ enhanced liquidity features relative to other debt securities, new issuances actually reduce liquidity in the market.¹⁰ Thus, while I will continue to use the phrase “monetary services” going forward for consistency, it seems that the Treasury is primarily supplying safety to the markets, not necessarily liquidity. Further exploration of this specific point is left to future research.

5.3 Further Analysis

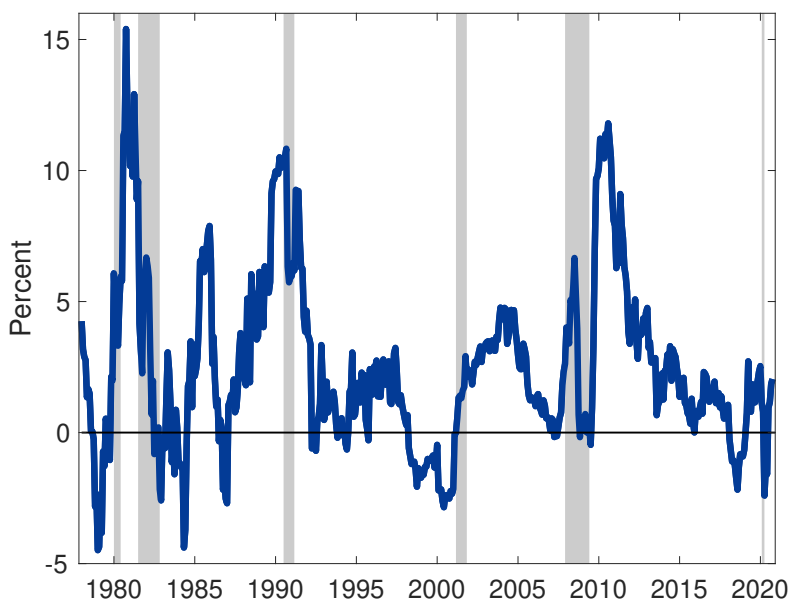


Figure 8: Year-over-year Growth of Fiscal Monetary Services

For a more intuitive and readable view of the growth rate of these monetary services, Figure 8 presents the spread in the year-over-year growth rates constructed from the index in Figure 5. One of the larger and more sustained spreads in recent years came with the combination of the American Recovery and Reinvestment Act of 2009 (ARRA) and the European debt crisis from January 2009, through about April 2013. The additional quantity of Treasury debt, combined with the rush to safety from European bond markets caused the monetary services of the underlying Treasury debt to outgrow the pure quantity being created for a pronounced time. The “dash for cash” can also be seen during the months of the COVID recession and the months immediately following. This crisis can be seen as a sharp decrease in the demand for monetary services bonds

¹⁰ Additionally, these are the monetary services of the entire Treasury portfolio, which not only includes Bills, but also the less-liquid Notes and Bonds in both nominal and TIPS form.

provided. The increase in monetary services can also be seen in the late-1980s and early-1990s as the savings-and-loan crisis burned through the US economy. This measure, and the events surrounding its peaks and troughs, lend additional credence to it capturing the monetary services—particularly safety—of US Treasury debt.

5.4 Fiscal Capacity Considerations

In Section 3.3 I showed that the total value of the monetary services aggregate M_t expands the fiscal capacity to borrow. Incorporating the aggregate’s price dual from (21) provides the measure of the total value shown in Figure 9. The data does not go back far enough to know whether the higher values in the late 1970s are a one-off spike or a sustained trend. The sudden decrease in this index in the 1989–1990 period comes from a sudden decrease in the price of obtaining these monetary services. While the reasoning for this dramatic shift is beyond the scope of this paper, this does generally align with some later estimates of the Great Moderation as well as the surge in the information technology realm.¹¹ One could imagine that the decrease in general market volatility, coupled with an increase in the available information about other financial assets, decimated the Treasury’s comparative advantage in the financial world.

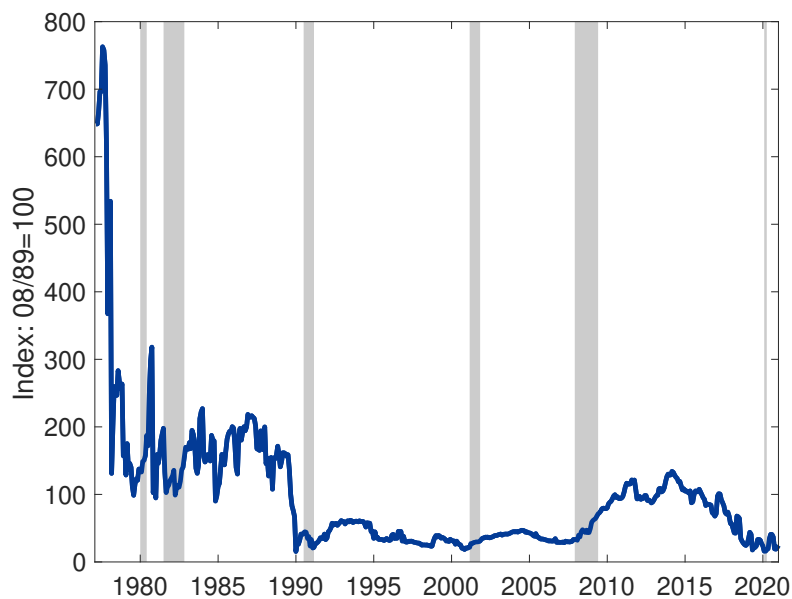


Figure 9: Value of Fiscal Monetary Services

The value of the monetary services aggregate was relatively stable until the financial

¹¹ While most of the work on the Great Moderation centers around 1984:Q4 as the break point, Stock and Watson (2003) find a relatively wide 95% confidence interval that expands all the way to 1989:Q4. The first web browser was also introduced in 1990, providing information in an easy-to-use format to the masses.

crisis of 2007-2009, when that value increased approximately 378 percent from January 2007 to January 2014. This increase in the fiscal capacity would help explain the lack of inflationary pressure during that period of expansion despite increases in federal spending over that time. The stimulus passed during the 2020 pandemic, however, did not provide the same increase in total value, as the private sector moved towards pure liquidity (“dash for cash”) in the face of lockdowns. In that type of environment, even Treasury securities do not provided the needed liquidity and safety attributes.

6 Fiscal Monetary Services and Inflation

The foundational intrigue of Brunnermeier *et al.* (2020, 2022) was in relation to the fiscal theory of the price level (FTPL). That is, to what degree does fiscal policy, and its debt specifically, influence inflation rates and the price level? While it has been difficult to find a link between the principal value of outstanding fiscal debt and inflation, in this section I again consider a Jordà (2005) linear projection model to explore the impact of these fiscal monetary services on inflation. Given the myriad of likely inflationary channels, estimating impulse response functions via local projections allows me more flexibility to incorporate more variables while simultaneously providing more protection against omitted variable bias.

Impulse response functions are estimated using a model with the same form as (23). All the data in this analysis is monthly in frequency and—after adjustments for data availability and lag order in both the identification and estimation steps—covers the 1986:3–2019:12 period. The dependent variable z_{t+h} is the year-over-year growth rate of the personal consumption expenditures chain-type price index, excluding food and energy (core PCE inflation). Given the potential for noise in month-over-month measures, using a year-over-year growth rate should provide a better perspective. The control variables y_t include lags of the unemployment rate, the 12-month ahead inflation expectations as measured by the University of Michigan’s Surveys of Consumers, the effective federal funds rate, the non-cyclical rate of unemployment, core PCE inflation, the year-over-year inflation rate of end-use import prices adjusted for their share of GDP, the year-over-year growth rate of spot oil prices, and the growth rate of the simple-sum debt aggregate. Dummy variables indicating recessions and the zero lower bound are also included. As is suggested by Kilian and Kim (2011), the lag order is determined at each horizon, based on AIC, and with a maximum lag order of six.¹²

Exogenous shocks to the year-over-year growth rate of fiscal monetary services shown in Figure 8 are derived using a model similar to (22). Here, the control variables X_t include the year-over-year changes in the Wilshire 5000 stock price index, trade-weighted US dollar index, index of consumer sentiment from the University of Michigan’s Surveys

¹² They also note that the models fit better when the maximum lag order is restricted. Allowing the maximum lag order to be considerably higher does not fundamentally change the results.

of Consumers, and dependent variable. Two lags are considered based on AIC. The residuals ϵ_t are the estimated exogenous shocks to the growth of fiscal monetary services.

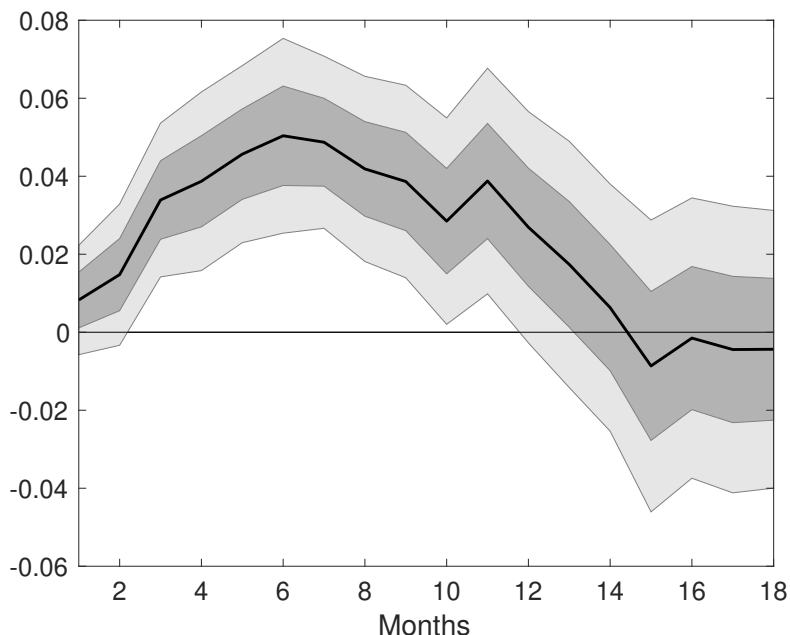


Figure 10: Inflationary Response to Fiscal Monetary Services Shock

Figure 10 presents the impact of a one-percentage-point increase in fiscal monetary services on the core PCE inflation rate. After a two-month lag, the year-over-year inflation rate peaks between four and five basis points and the statistical significance of the shock persists for about eight months. Given that the average year-over-year growth rate in the sample is approximately 2.5 percent and has frequently reached the 5-10 percent range, the impact of fiscal monetary services is both economically large and statistically significant. This result therefore provides evidence in favor of the FTPL via a monetary service channel.

7 Conclusion

The stock of outstanding US government debt has consistently increased since the early 2000s, yet interest rates have remained near historic lows and prices have only recently shown signs of accelerating. Prior to that, borrowing costs continuously fell as government debt continuously rose. This has prompted many to reconsider both the traditional views of government bonds and fiscal budgetary constraints (e.g. Krishnamurthy and Vissing-Jorgensen, 2012; Brunnermeier *et al.*, 2020).

In this paper, I use the idea that government securities provide monetary services to the holders to construct a Fisher ideal index of marketable US Treasury debt. I show

that the value of these monetary services increases the fiscal capacity of the government in a simple partial equilibrium model with short- and long-term debt. I then use this constructed metric, in comparison to the simple sum aggregate, to uncover the growth rate of these fiscally-provided monetary services and show that an increase has an inflationary impact that is persistently positive and both statistically and economically significant. These results suggest that there is a monetary services channel to the FTPL. That is, while finding evidence linking the stock of outstanding principal to the price level has been difficult to come by, my findings offer another plausible channel.

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A Representative Household Solution

Combining (2) and (3)

$$\left(r_t^L - r_t^{L,n}\right) B_t^L = (1 - \alpha) \left(r_{t-1}^L - r_t^{L,n}\right) B_{t-1}^L, \quad (\text{A.1})$$

(4) and (5)

$$(R_t - R_t^n) K_t = (1 - \delta) (R_{t-1} - R_t^n) K_{t-1}, \quad (\text{A.2})$$

and (6), (2), and (4)

$$\begin{aligned} B_t + B_t^L - (1 - \alpha) B_{t-1}^L + p_t c_t + K_t - (1 - \delta) K_{t-1} &= (\delta^* + R_{t-1}) K_{t-1} \\ &+ (1 + r_{t-1}) B_{t-1} + (\alpha + r_{t-1}^L) B_{t-1}^L + (1 - \tau_t) W_t l_t + \int_0^1 \Pi_t(s) ds. \end{aligned} \quad (\text{A.3})$$

A.1 Household's Problem

(7) subject to (8), (A.1), (A.2), and (A.3). Choosing $\{B_t, B_t^L, c_t, K_t, R_t, r_t^L, l_t, M_t\}$

A.2 Bellman Equation

$$\begin{aligned} &\mathbb{V}(B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L) \\ &= \max \left\{ u(c_t) + v \left(\frac{M_t}{P_t} \right) + x(l_t) + \beta \mathbb{E}_t [\mathbb{V}(B_t, B_t^L, K_t, R_t, r_t^L)] \right. \\ &+ \frac{\mu_{1,t}}{p_t} \left[(\delta^* + R_{t-1}) K_{t-1} + (1 + r_{t-1}) B_{t-1} + (\alpha + r_{t-1}^L) B_{t-1}^L + (1 - \tau_t) W_t l_t \right. \\ &+ \int_0^1 \Pi_t(s) ds - B_t - B_t^L + (1 - \alpha) B_{t-1}^L - p_t c_t - K_t + (1 - \delta) K_{t-1} \left. \right] \\ &+ \frac{\mu_{2,t}}{p_t} \left[\left[\lambda^{\frac{1}{\sigma}} B_t^{\frac{\sigma-1}{\sigma}} + (1 - \lambda)^{\frac{1}{\sigma}} B_t^L \frac{\sigma-1}{\sigma} \right]^{\frac{\sigma}{1-\sigma}} - M_t \right] \\ &+ \frac{\mu_{3,t}}{p_t} \left[(1 - \delta) (R_{t-1} - R_t^n) K_{t-1} - (R_t - R_t^n) K_t \right] \\ &\left. + \frac{\mu_{4,t}}{p_t} \left[(1 - \alpha) \left(r_{t-1}^L - r_t^{L,n} \right) B_{t-1}^L - \left(r_t^L - r_t^{L,n} \right) B_t^L \right] \right\} \quad (\text{A.4}) \end{aligned}$$

A.3 First Order Conditions

$$u'(c_t) = \mu_{1,t} \quad (\text{A.5})$$

$$v' \left(\frac{M_t}{p_t} \right) = \mu_{2,t} \quad (\text{A.6})$$

$$x'(l_t) + \mu_{1,t} \frac{W_t}{p_t} (1 - \tau_t) = 0 \quad (\text{A.7})$$

$$\beta \mathbb{E}_t [\mathbb{V}_1 (B_t, B_t^L, K_t, R_t, r_t^L)] = \frac{\mu_{1,t}}{p_t} - \frac{\mu_{2,t}}{p_t} \left(\frac{\lambda M_t}{B_t} \right)^{\frac{1}{\sigma}} \quad (\text{A.8})$$

$$\beta \mathbb{E}_t [\mathbb{V}_2 (B_t, B_t^L, K_t, R_t, r_t^L)] = \frac{\mu_{1,t}}{p_t} - \frac{\mu_{2,t}}{p_t} \left(\frac{(1 - \lambda) M_t}{B_t^L} \right)^{\frac{1}{\sigma}} + \frac{\mu_{4,t}}{p_t} (r_t^L - r_t^{L,n}) \quad (\text{A.9})$$

$$\beta \mathbb{E}_t [\mathbb{V}_3 (B_t, B_t^L, K_t, R_t, r_t^L)] = \frac{\mu_{1,t}}{p_t} + \frac{\mu_{3,t}}{p_t} (R_t - R_t^n) \quad (\text{A.10})$$

$$\beta \mathbb{E}_t [\mathbb{V}_4 (B_t, B_t^L, K_t, R_t, r_t^L)] = \frac{\mu_{3,t}}{p_t} K_t \quad (\text{A.11})$$

$$\beta \mathbb{E}_t [\mathbb{V}_5 (B_t, B_t^L, K_t, R_t, r_t^L)] = \frac{\mu_{4,t}}{p_t} B_t^L \quad (\text{A.12})$$

A.4 Bienveniste-Scheinkman Conditions

$$\mathbb{V}_1 (B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L) = \frac{\mu_{1,t}}{p_t} (1 + r_{t-1}) \quad (\text{A.13})$$

$$\mathbb{V}_2 (B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L) = \frac{\mu_{1,t}}{p_t} (1 + r_{t-1}^L) + \frac{\mu_{4,t}}{p_t} (1 - \alpha) (r_{t-1}^L - r_t^{L,n}) \quad (\text{A.14})$$

$$\mathbb{V}_3 (B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L) = \frac{\mu_{1,t}}{p_t} (\delta^* - \delta + 1 + R_{t-1}) + \frac{\mu_{3,t}}{p_t} (1 - \delta) (R_{t-1} - R_t^n) \quad (\text{A.15})$$

$$\mathbb{V}_4 (B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L) = \left(\frac{\mu_{1,t}}{p_t} + (1 - \delta) \frac{\mu_{3,t}}{p_t} \right) K_{t-1} \quad (\text{A.16})$$

$$\mathbb{V}_5 (B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L) = \left(\frac{\mu_{1,t}}{p_t} + (1 - \alpha) \frac{\mu_{4,t}}{p_t} \right) B_{t-1}^L \quad (\text{A.17})$$

A.5 Optimality Conditions

$$u'(c_t) = \mu_{1,t} \quad (\text{A.18})$$

$$v' \left(\frac{M_t}{p_t} \right) = \mu_{2,t} \quad (\text{A.19})$$

$$x'(l_t) + \mu_{1,t} \frac{W_t}{p_t} (1 - \tau_t) = 0 \quad (\text{A.20})$$

$$\frac{\mu_{1,t}}{p_t} - \frac{\mu_{2,t}}{p_t} \left(\frac{\lambda M_t}{B_t} \right)^{\frac{1}{\sigma}} = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{p_{t+1}} (1 + r_t) \right] \quad (\text{A.21})$$

$$\begin{aligned} \frac{\mu_{1,t}}{p_t} - \frac{\mu_{2,t}}{p_t} \left(\frac{(1-\lambda)M_t}{B_t^L} \right)^{\frac{1}{\sigma}} + \frac{\mu_{4,t}}{p_t} (r_t^L - r_t^{L,n}) \\ = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{p_{t+1}} (1 + r_t^L) + \frac{\mu_{4,t+1}}{p_{t+1}} (1 - \alpha) (r_t^L - r_{t+1}^{L,n}) \right] \end{aligned} \quad (\text{A.22})$$

$$\begin{aligned} \frac{\mu_{1,t}}{p_t} + \frac{\mu_{3,t}}{p_t} (R_t - R_t^n) \\ = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{p_{t+1}} (\delta^* - \delta + 1 + R_t) + \frac{\mu_{3,t+1}}{p_{t+1}} (1 - \delta) (R_t - R_{t+1}^n) \right] \end{aligned} \quad (\text{A.23})$$

$$\frac{\mu_{3,t}}{p_t} K_t = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{p_{t+1}} + (1 - \delta) \frac{\mu_{3,t+1}}{p_{t+1}} \right] K_t \quad (\text{A.24})$$

$$\frac{\mu_{4,t}}{p_t} B_t^L = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{p_{t+1}} + (1 - \alpha) \frac{\mu_{4,t+1}}{p_{t+1}} \right] B_t^L \quad (\text{A.25})$$

B Derivation of the User Costs

Combining (11) and (12) yields:

$$1 = \beta \mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n + \delta^* - \delta - (1 - \delta) \gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right] \quad (\text{B.1})$$

Substituting this for the 1 in (9) and rearranging yields the marginal benefit/marginal cost equilibrium:

$$\gamma_{2,t} \left(\frac{\lambda M_t}{B_t} \right)^{\frac{1}{\sigma}} = \beta \mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n + \delta^* - \delta - (1 - \delta) \gamma_{3,t+1} \Delta R_{t+1}^n - r_t \right\} \right] \quad (\text{B.2})$$

Now dividing both sides by (B.1) converts the right side to the standard user cost form

$$\gamma_{2,t} \left(\frac{\lambda M_t}{B_t} \right)^{\frac{1}{\sigma}} = \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n + \delta^* - \delta - (1 - \delta) \gamma_{3,t+1} \Delta R_{t+1}^n - r_t \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n + \delta^* - \delta - (1 - \delta) \gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right]} \quad (\text{B.3})$$

Decoupling $\gamma_{2,t}$ shows that the right hand side is the marginal cost of holding the short-term asset, expressed in terms of utility.

$$v' \left(\frac{M_t}{p_t} \right) \left(\frac{\lambda M_t}{B_t} \right)^{\frac{1}{\sigma}} = u'(c_t) \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n + \delta^* - \delta - (1 - \delta) \gamma_{3,t+1} \Delta R_{t+1}^n - r_t \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n + \delta^* - \delta - (1 - \delta) \gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right]} \quad (\text{B.4})$$

If it is assumed that that $\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}}$ is independent of the one-period returns in brackets, this expression simplifies even further to

$$\eta_t = \frac{R_t^n + \delta^* - \delta - (1 - \delta)\mathbb{E}_t[\gamma_{3,t+1}\Delta R_{t+1}^n] - r_t}{1 + R_t^n + \delta^* - \delta - (1 - \delta)\mathbb{E}_t[\gamma_{3,t+1}\Delta R_{t+1}^n]}, \quad (\text{B.5})$$

where η_t equals the left-hand side of (B.3) and represents the user cost of holding the short term asset for one period.

Beginning the same procedure from (10) will provide the analogous user cost for a long-term security

$$\eta_t^L = \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n + \delta^* - \delta - (1 - \delta)\gamma_{3,t+1}\Delta R_{t+1}^n - r_t^{L,n} + (1 - \alpha)\gamma_{4,t+1}\Delta r_{t+1}^{L,n} \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n + \delta^* - \delta - (1 - \delta)\gamma_{3,t+1}\Delta R_{t+1}^n \right\} \right]}, \quad (\text{B.6})$$

and again assuming independence as above yields

$$\eta_t^L = \frac{R_t^n + \delta^* - \delta - (1 - \delta)\mathbb{E}_t[\gamma_{3,t+1}\Delta R_{t+1}^n] - r_t^{L,n} + (1 - \alpha)\mathbb{E}_t[\gamma_{4,t+1}\Delta r_{t+1}^{L,n}]}{1 + R_t^n + \delta^* - \delta - (1 - \delta)\mathbb{E}_t[\gamma_{3,t+1}\Delta R_{t+1}^n]}. \quad (\text{B.7})$$

Since we're dealing with long-term assets here, the period-by-period user cost incorporates both the expected one-period payouts as well as the expected capital gains/loses. The capital gains are incorporated via the expected change in the one-period payouts, scaled by the expected future price of the asset.

C Derivation of the Government's Budget Constraint

Starting with ??, and given (B.5) and (B.7) and some rearranging, we can simplify the above equation to

$$\begin{aligned} \frac{B_{t-1}(1 + r_{t-1}) + B_{t-1}^L(1 + r_{t-1}^L)}{p_t} &= t_t - g_t \\ &+ \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{B_t(1 + r_t) + B_t^L(1 + r_t^L)}{p_{t+1}} \right] + \eta_t \frac{B_t}{p_t} + \eta_t^L \frac{B_t^L}{p_t} \\ &+ \left\{ \beta \mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left(r^{L,n} - (1 - \alpha)\gamma_{4,t+1}\Delta r_{t+1}^{L,n} \right) \right] - \beta \mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \right] r_t^L \right\} \frac{B_t^L}{p_t} \end{aligned} \quad (\text{C.1})$$

where η_t and η_t^L are the user costs of short-term and long-term debt as described in (B.5) and (B.7), respectively. Next, I use (B.1) to add zero into the term in brackets

and then divide the left section of that term by one

$$\begin{aligned}
& \frac{B_{t-1}(1+r_{t-1}) + B_{t-1}^L(1+r_{t-1}^L)}{p_t} = t_t - g_t \\
& + \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1} B_t(1+r_t) + B_t^L(1+r_t^L)}{\mu_{1,t} p_{t+1}} \right] + \eta_t \frac{B_t}{p_t} + \eta_t^L \frac{B_t^L}{p_t} \\
& + \left\{ - \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n + \delta^* - \delta - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n - r_t^{L,n} + (1-\alpha)\gamma_{4,t+1} \Delta r_{t+1}^{L,n} \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n + \delta^* - \delta - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right]} \right. \\
& \qquad \qquad \qquad \left. - \beta \mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \right] (1+r_t^L) \right\} \frac{B_t^L}{p_t}. \quad (\text{C.2})
\end{aligned}$$