Description of the Math Accuplacer Test

Union Building Testing Center Room 323: 801-626-7945
Davis Campus Testing Center Room 215: 801-395-3495

For other locations, schedules and additional info go to http://weber.edu/TestingCenter/accuplacer.html

Cost: Math: $10.00 (Score Reprints: $10.00)

Bring to Testing Center: Picture ID and W number (The W number is the ID assigned to a student when they are admitted. It is printed on Admissions and Registration material.) Calculators are NOT allowed but scratch paper will be provided

Arithmetic Portion 17 questions 120 point maximum

1. Score of 20-92 places student in **Math 0950**

- **Operations with whole numbers and fractions** – addition, subtraction, multiplication, division, recognizing equivalent fractions and mixed numbers.
- **Operations with decimals and percents** – additions, subtraction, multiplication, and division percent problems, decimal recognition, fraction percent equivalencies, and estimation problems.
- **Applications and problem solving** – rate, percent, and measurement problems, geometry problems, distribution of a quantity into its fractional parts.

Elementary Algebra Portion 12 questions 120 point maximum

1. Score of 0-81 places student in **Math 0990**
2. Score of 82 or higher places student in **Math 1010**

- **Operations with integers and rational numbers** – computation with integers and negative rationals, the use of absolute values, and ordering.
- **Operations with algebraic expressions** – evaluations of simple formulas, expressions, and adding, subtracting monomials and polynomials, the evaluation of positive rational roots and exponents, simplifying algebraic fractions, and factoring.
- **Equations solving, inequalities, and word problems** – solving verbal problems presented in algebraic context, geometric reasoning, the translation of written phrases into algebraic expressions, graphing.

College Level Math Portion 20 questions 120 point maximum

1. Score of 0-49 places student in **Math 1010**
2. Score of 50 to 69 places student in **any QL Math course or Math 1060**
3. Score of 70 to 89 **fulfills QL requirement and student is able to register for courses that require Math 1050 as a prerequisite**
4. Score of 90 or higher places student in **Math 1210**

- **Algebraic operations** – simplifying rational algebraic expressions, factoring and expanding polynomials, manipulating roots and exponents.
- **Solutions of equations and inequalities** – the solution of linear and quadratic equations by factoring, expanding polynomials, manipulating roots and exponents.
- **Coordinate geometry** – plane geometry, the coordinate plane, straight lines, conics, sets of points in a plane, graphs of algebraic functions.
- **Application and other algebra topics** – complex numbers, series and sequences, determinants, permutations, combinations, fractions, word problems.
- **Functions and trigonometry** – polynomial, algebraic, exponential, logarithmic, trigonometric functions.

CENTERS

Distance Accuplacer Proctor Request

To take the Accuplacer in your local community:

1. **You must be far enough away from Weber State University to qualify for the Distance Accuplacer.**
   Outside of Utah qualifies, but inside Utah the test will not be sent to any site between the Utah/Idaho state line and the Point of the Mountain with the exception of Utah State University in Logan, UT.

2. **Contact a local college or university testing center (High Schools are not acceptable).**
   - Ask if they will proctor your assessment test.
   - The testing center will need a computer with Internet access.
   - Be sure the chosen proctor is willing to give a test lasting anywhere from 1-2-+ hours.

3. **Register for your desired test (buttons in right-column).**

4. **Your request will be automatically emailed to our office.**
   Your test request can take 2 to 7 business days to process. We will notify the proctor with the needed information for your assessment test.

5. **There is a $10.00 fee per test, payable to Weber State University.**
   This fee must be paid before the test can be setup. Once the test is sent to the proctor the payment is processed and no refund is given.

6. **You will be notified by email once your test has been sent to the proctor.**

7. **After you have been notified that your placement test has been sent to the proctor, you will have until the end of the month to take your test.**
   After that time the information sent to the proctor will no longer be valid.

Distance Accuplacer Proctor Request Website:
http://www.weber.edu/TestingCenter/distance-accuplacer.html

WEBER STATE UNIVERSITY
ASSESSMENT AND PLACEMENT STANDARDS
(Original Date: 10/15/05 Revision Date: 02/01/13)

This document contains information about current Quantitative Literacy and Composition course placement information based on student ACT/SAT, AP, and Accuplacer scores.

QUANTITATIVE LITERACY (QL) REQUIREMENT

HOW DO YOU FULFILL YOUR QUANTITATIVE LITERACY (QL) REQUIREMENT?

There are 5* ways to fulfill your QL requirement:

- Earn "C" or better in MATH 1030, 1040, 1050, or 1080 or
- Earn "C" or better in any math course for which either MATH 1050 or 1080 is a prerequisite or
- Score 70 or higher on College Level Math (CLM) in Accuplacer or
- Score 3 or higher on AP Calculus exam or
- Score 3 or higher on AP Statistics exam or

Before choosing one of these 5 options, consult the WSU Catalog or your major advisor. Many programs will require you to take a specific QL course.

Depending on your placement score, you may need to take one or more prerequisite courses before enrolling in a QL course. Your placement score indicates where in the QL sequence you need to begin. The QL prerequisite sequence is as follows:

- MATH 0950 → MATH 0990[◇] → MATH 1010 → MATH QL course

Important note: Once you pass a prerequisite math course, the prerequisite is valid for only two years before it expires.

WHICH WSU COURSE DO I PLACE INTO BASED ON MY MATH ACT SCORE?

ACT[†] Math Score [expires after 2 years]	WSU Course Placement
Didn't take ACT or SAT	Take Math Accuplacer for course placement
ACT 22 or lower	Take Math Accuplacer for course placement
ACT 23 or higher	MATH 1030, 1040, 1050, 1060, or 1080 Note: MATH 1060 does NOT satisfy the QL requirement

WHICH WSU COURSE DO I PLACE INTO BASED ON MY MATH ACCUPLACER SCORE?

Accuplacer° Score [expires after 2 years]	WSU Course Placement
Arithmetic (AR) 20-92	MATH 0950
Elementary Algebra (EA) 0-81	MATH 0990[◇]
Elementary Algebra (EA) 82 or higher College Level Math (CLM) 0-49	MATH 1010
College Level Math (CLM) 50 or higher	MATH 1030, 1040, 1050, 1060, or 1080 Note: MATH 1060 does NOT satisfy QL requirement
College Level Math (CLM) 70 or higher	Fulfills QL requirement and MATH 1050 prerequisite[‡]
College Level Math (CLM) 90 or higher	MATH 1210 [only required for some majors]

*If you earned a "C" or better in PHIL 2200 between Spring 2007 and Spring 2013 and you are declared in a catalog year between Spring 2007 and Spring 2013, your QL requirement is also met.

[◇]Math 0990 is equivalent to Math 0960 (course number changed Summer 2011).

[†]If you took the SAT, rather than the ACT, consult Admissions at 801-626-6743, weber.edu/admissions or the Student Success Center at 801-626-6752 for your placement.

[°]You may take the Math Accuplacer exam up to 2 times within a 1 year period. After taking the Math Accuplacer twice in 1 year, you may request to take an approved alternative math placement test through the Testing Center. This test will allow you to place up to, but not out of, QL.

[‡]If you score CLM 70 or higher, your QL requirement is fulfilled regardless of when the exam was taken; however, the score expires as a prerequisite for subsequent math courses after 2 years.

Arithmetic Sample Questions

This test measures your ability to perform basic arithmetic operations and to solve problems that involve fundamental arithmetic concepts. There are 17 questions on the Arithmetic tests, divided into three types.

• Operations with whole numbers and fractions: Topics included in this category are addition, subtraction, multiplication, division, recognizing equivalent fractions and mixed numbers, and estimating.

• Operations with decimals and percents: Topics include addition, subtraction, multiplication, and division with decimals. Percent problems, recognition of decimals, fraction and percent equivalencies, and problems involving estimation are also given.

• Applications and problem solving: Topics include rate, percent, and measurement problems; simple geometry problems; and distribution of a quantity into its fractional parts.

1. 2.75 + .003 + .158 =

   A. 4.36
   B. 2.911
   C. 0.436
   D. 2.938

2. 7.86 × 4.6 =

   A. 36.156
   B. 36.216
   C. 351.56
   D. 361.56

3. $\frac{7}{20}$ =

   A. .035
   B. 0.858
   C. 0.35
   D. 3.5

4. Which of the following is the least?

   A. 0.105
   B. 0.501
   C. 0.015
   D. 0.15

5. All of the following are ways to write 25 percent of N EXCEPT:

   A. 0.25 N
   B. 25N/100
   C. 1/4 N
   D. 25 N

6. Which of the following is closest to 27.8 x 9.6?

   A. 280
   B. 300
   C. 2,800
   D. 3,000

7. A soccer team played 160 games and won 65% of them. How many games did it win?

   A. 94
   B. 104
   C. 114
   D. 124

8. 32 is 40% of what number?

   A. 12.8
   B. 128
   C. 80
   D. 800

Arithmetic – cont'

9. Three people who work full-time are to work together on a project, but their total time on the project is to be equivalent to that of only one person working full-time. If one of the people is budgeted for one-half of his time to the project and a second person for one-third of her time, what part of the third worker's time should be budgeted to this project?

A. 1/3
B. 3/5
C. 1/6
D. 1/8

10. 3 1/3 – 2 2/5 =

A. 1 1/2
B. 1/15
C. 14/15
D. 1 1/15

Elementary Algebra Sample Questions

A total of 12 questions of three types are administered in this test.

• The first type involves operations with integers and rational numbers, and includes computation with integers and negative rationals, the use of absolute values, and ordering.

• The second type involves operations with algebraic expressions using evaluation of simple formulas and expressions, and adding and subtracting monomials and polynomials. Questions involve multiplying and dividing monomials and polynomials, the evaluation of positive rational roots and exponents, simplifying algebraic fractions, and factoring.

• The third type of question involves translating written phrases into algebraic expressions and solving equations, inequalities, word problems, linear equations and inequalities, quadratic equations (by factoring), and verbal problems presented in an algebraic context.

1. If A represents the number of apples purchased at 15 cents each, and B represents the number of bananas purchased at 10 cents each, which of the following represents the total value of the purchases in cents?

A. A+B
B. 25(A+B)
C. 10A + 15B
D. 15A + 10B

2. $\sqrt{2} \times \sqrt{15} =$

A. 17
B. 30
C. $\sqrt{30}$
D. $\sqrt{17}$

Elementary Algebra – cont'

3. What is the value of the expression $2x^2 + 3xy - 4y^2$ when x = 2 and y = -4 ?

A. -80
B. 80
C. -32
D. 32

4. In the figure below, both circles have the same center, and the radius of the larger circle is R. If the radius of the smaller circle is 3 units less than R, which of the following represents the area of the shaded region?

A. πR^2
B. $\pi(R\text{-}3)^2$
C. $\pi R^2 - \pi \times 3^2$
D. $\boldsymbol{\pi} R^2 - \boldsymbol{\pi}(R\text{-}3)^2$

5. $(3x - 2y)^2 =$

A. $9x^2 - 4y^2$
B. $9x^2 + 4y^2$
C. $9x^2 + 4y^2 - 6xy$
D. $9x^2 + 4y^2 - 12xy$

6. If $x > 2$, then $\frac{x^2-x-6}{x^2-4} =$

A. $\frac{x-3}{2}$
B. $\frac{x-3}{x-2}$
C. $\frac{x-3}{x+2}$
D. $\frac{3}{2}$

7. If x 2, then $\frac{4-(-6)}{-5} =$

A. $\frac{2}{5}$
B. $-\frac{2}{5}$
C. 2
D. -2

8. If $2x - 3(x+4) = -5$, then $x =$

A. 7
B. -7
C. 17
D. -17

9. -3(5 - 6) - 4(2 - 3) =

A. -7
B. 7
C. -1
D. 1

10. Which of the following expressions is equivalent to $20 - \frac{4}{5}x \geq 16$?

A. $x \leq 5$
B. $x \geq 5$
C. $x \geq 32^1/_2$
D. $x \leq 32^1/_2$

College Level Math Sample Questions

The College-Level Mathematics test measures your ability to solve problems that involve college-level mathematics concepts. There are six content areas measured on this test: (a) Algebraic Operations, (b) Solutions of Equations and Inequalities, (c) Coordinate Geometry, (d) Applications and other Algebra Topics, (e) Functions, and (f) Trigonometry. The Algebraic Operations content area includes the simplification of rational algebraic expressions, factoring and expanding polynomials, and manipulating roots and exponents. The Solutions of Equations and Inequalities content area includes the solution of linear and quadratic equations and inequalities, systems of equations, and other algebraic equations. The Coordinate Geometry content area presents questions involving plane geometry, the coordinate plane, straight lines, conics, sets of points in the plane, and graphs of algebraic functions. The Functions content area includes questions involving polynomial, algebraic, exponential, and logarithmic functions. The Trigonometry content area includes trigonometric functions. The Applications and other Algebra Topics content area contains complex numbers, series and sequences, determinants, permutations and combinations, factorials, and word problems. A total of 20 questions are administered on this test.

1. $2^{5/2} - 2^{3/2} =$

A. $2^{1/2}$
B. 2
C. $2^{3/2}$
D. $2^{5/3}$
E. 2^2

2. If $a \neq b$, and $\frac{1}{x} + \frac{1}{a} = \frac{1}{b}$, then $x =$

A. $\frac{1}{b} - \frac{1}{a}$
B. b-a
C. $\frac{1}{ab}$
D. $\frac{a-b}{ab}$
E. $\frac{ab}{a-b}$

3. If $3x^2 - 2x + 7 = 0$, then $\left(x - \frac{1}{3}\right)^2 =$

A. $\frac{20}{9}$
B. $\frac{7}{9}$
C. $-\frac{7}{9}$
D. $-\frac{8}{9}$
E. $-\frac{20}{9}$

4. The graph of which of the following equations is a straight line parallel to the graph of $y = 2x$?

A. $4x - y = 4$
B. $2x - 2y = 2$
C. $2x - y = 4$
D. $2x + y = 2$
E. $x - 2y = 4$

5. An equation of the line that contains the origin and the point (1,2) is

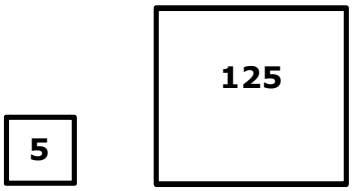
A. $y = 2x$
B. $2y = x$
C. $y = x - 1$
D. $y = 2x + 1$
E. $\frac{y}{2} = x - 1$

6. An apartment building contains 12 units consisting of one- and two-bedroom apartments that rent for \$360 and \$450 per month, respectively. When all units are rented, the total monthly rental is \$4,950. What is the number of two-bedroom apartments?

A. 3
B. 4
C. 5
D. 6
E. 7

College Level Math – cont'

7. If the two square regions in the figures below have the respective areas indicated in square yards, how many yards of fencing are needed to enclose the two regions?



A. $4\sqrt{130}$

B. $20\sqrt{10}$

C. $24\sqrt{5}$

D. 100

E. $104\sqrt{5}$

8. If $\log_{10} x = 3$, then $x =$

A. 3^{10}

B. 1,000

C. 30

D. $\frac{10}{3}$

E. $\frac{3}{10}$

9. If f(x) = 2x + 1 and g(x) = $\frac{x-1}{2}$, then f(g(x)) =

A. x

B. $\frac{x-1}{4x+2}$

C. $\frac{4x+2}{x-1}$

D. $\frac{5x+1}{2}$

E. $\frac{(2x+1)(x-1)}{2}$

10. If θ is an acute angle and $\sin\theta = \frac{1}{2}$, then $\cos\theta =$

A. -1

B. 0

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$

E. 2

Arithmetic: 1)B 2)A 3)C 4)C 5)D 6)A 7)B 8)C 9)C 10)C

Elementary Algebra: 1)D 2)C 3)A 4)D 5)D 6)B 7)D 8)B 9)B 10)A

College Level Math: 1)C 2)E 3)E 4)C 5)A 6)E 7)C 8)B 9)A 10)D

Fraction Rules

Special Fractions

1. $\frac{b}{1}$ simplifies to b.
2. $\frac{1}{b}$ does not simplify any further.
3. $\frac{0}{b}$ simplifies to 0.
4. $\frac{b}{0}$ is undefined.

Negative Fractions

1. $-\frac{a}{b}$ is the same as $\frac{-a}{b}$ and $\frac{a}{-b}$
2. $\frac{-a}{-b}$ simplifies to $\frac{a}{b}$
3. $-\frac{a}{b}$ is NOT the same as $\frac{-a}{-b}$

Addition

1. $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$
2. $a+\frac{b}{c}=\frac{ac}{c}+\frac{b}{c}=\frac{ac+b}{c}$
3. $\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$

Subtraction

1. $\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$
2. $a-\frac{b}{c}=\frac{ac}{c}-\frac{b}{c}=\frac{ac-b}{c}$
3. $\frac{a}{b}-c=\frac{a}{b}-\frac{bc}{b}=\frac{a-bc}{b}$
4. $\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}$

Multiplication

1. $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$
2. $a\cdot\frac{b}{c}=\frac{a}{1}\cdot\frac{b}{c}=\frac{ab}{c}$
3. $\frac{a}{b}\cdot c=\frac{a}{b}\cdot\frac{c}{1}=\frac{ac}{b}$

Division

1. $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$
2. $\frac{\frac{a}{b}}{c}=\frac{\frac{a}{b}}{\frac{c}{1}}=\frac{a}{b}\cdot\frac{1}{c}=\frac{a}{bc}$
3. $\frac{a}{\frac{b}{c}}=\frac{\frac{a}{1}}{\frac{b}{c}}=\frac{a}{1}\cdot\frac{c}{b}=\frac{ac}{b}$

Cancellation ($a \neq 0$, $b \neq 0$, $c \neq 0$)

1. $\frac{a}{a}$ cancels to 1
2. $\frac{ab}{ac}$ cancels to $\frac{b}{c}$
3. $\frac{a}{b}\cdot\frac{b}{c}$ cancels to $\frac{a}{c}$
4. $\frac{a}{b}\cdot\frac{c}{a}$ cancels to $\frac{c}{b}$
5. $a\cdot\frac{b}{a}$ cancels to b
6. $\frac{b}{a}\cdot a$ cancels to b

Decimal Rules

Comparing

1. Give the decimals the same number of places. (You can put zeros to the right of a decimal without changing its value.)

2. Compare each number from the decimal to the right, deciding which decimal is larger.

Addition

1. Line up the decimals with point under point.

2. Add each column and bring the decimal point straight down into each answer. Carry if necessary. You can carry to the left of the decimal point.

```
  12.924
+ 3.600
  16.524
```

Warning: Don't confuse the period at the end of a sentence with the decimal point.

Subtraction

1. Use zeros to give each number the same number of decimal places. Compare the decimals to decide which is larger.

2. Line up the numbers with point under point. Be sure to put the larger number on top.

3. Subtract, borrowing if necessary. Bring the decimal point straight down to the answer.

```
  18.200
- 6.008
  12.192
```

Division by a Decimal

To divide a number by a decimal, you must first change the divisor to a whole number.

1. Make the divisor a whole number by moving the point to the right as far as it will go.

   .45)$\overline{.900}$

2. Move the point in the dividend to the right the same number of places you moved the point in the divisor. You may need to add zeros to the dividend.

   45)$\overline{90.0}$

3. Bring the point up in the quotient directly above its new position in the dividend and divide.

   $\underline{2.0}$
   45)$\overline{90.0}$

90 divided by 45 = 2
is the same as .900 divided by.45 = 2

Division of Decimal by Whole Number

1. Put the point in the quotient directly above its position in the dividend.

2. Divide as you would for whole numbers.

Decimal Rules – cont.

Multiplication

1. Place one number under the other, lined up on the right side for easy multiplication. Multiply.

2. Count the number of decimal places in both numbers. (Decimal places are to the right of the decimal point.)

3. Counting from the right to the left put the total number of decimal places in your answer. Use zeros if you need more places than you have numbers.

37.7 x 2.8 =? --->

```
   37.7     ( 1 decimal place )
  x  2.8    ( 1 decimal place )
   3016
  +754
  105.56    (2 decimal places, move point 2 places left)
```

Rounding

1. Underline the digit in the place you are rounding off to.

2. If the digit to the right is 5 or more, add 1 to the underlined digit.

3. If the digit to the right is less than 5, leave the underlined digit as it is.

4. Drop the digits to the right of the underlined digit.

1.19 to the nearest tenth gives 1.2 (1.20).
1.545 to the nearest hundredth gives 1.55.
0.1024 to the nearest thousandth gives 0.102.
1.80 to the nearest one gives 2.
150.090 to the nearest hundred gives 200.
4499 to the nearest thousand gives 4000.

Estimating

Use estimating to help you check the placement of the decimal point. You could round 37.7 to 40 and 2.8 to 3. It's easy to multiply 3 x 40 so you know your answer should be close to 120.

Here's a "mental math" shortcut: When multiplying a number by a multiple of ten, just move the decimal point one space to the right for every zero.

10 x 0.6284 = 6.284 (1 zero, 1 space right)
100 x 0.6284 = 62.84 (2 zeroes, 2 spaces right)
1000 x 0.6284 = 628.4 (3 zeroes, 3 spaces right)
10,000 x 0.6284 = 6284 (4 zeroes, 4 spaces right)
100,000 x 0.6284 = 62,840 (5 zeroes, 5 spaces right)

Converting Fractions/Decimals/Percents Rules

A fraction to a decimal:

Divide the denominator (the bottom part) into the numerator (the top part):
$^{1}/4 = 1 \div 4.00 = 0.25$

A percent to a decimal:

Move the decimal point two places to the left. Then, drop the percent sign.
25% = 0.25

A fraction to a percent:

Multiply the fraction by 100 and reduce it. Then, attach a percent sign.
$^{1}/4 \times {}^{100}/1 = {}^{100}/4 = {}^{25}/1 = 25\%$

A decimal to a percent:

Move the decimal point two places to the right. Then, attach a percent sign.
0.25 = 25%

A decimal to a fraction:

Starting from the decimal point, count the decimal places. If there is one decimal place, put the number over 10 and reduce. If there are two places, put the number over 100 and reduce. If there are three places, put it over 1000 and reduce, and so on.
$0.25 = {}^{25}/100 = {}^{1}/4$

A percent to a fraction:

Put the number over 100 and reduce. Then, drop the percent sign.
$25\% = {}^{25}/100 = {}^{1}/4$

Percent Equation:

What percent of the total is the part? % x T = P

12% of the 200 students enrolled in freshman English earned a grade of "A" in the class. How many students earned an "A"?
Ex: 12% of 200 is what - Translate into an equation ("of" means "multiply"; "is" means "equal")
0.12 x 200 = *x* *(Change % to a decimal)*
x = 24 students earned an A

Percent Decrease/Increase:

Last year student employment jobs paid $7.25 per hour. This year student employment jobs are paying $8.45 per hour. What percent increase was given to student employment jobs?
1. Find the amount of the **increase**: $8.45 - $7.25 = **$1.20**
2. Which (hourly pay) **total** received an increase? The **$7.25** per hour got increased.
3. What % of the total was the increase?
$x \cdot 7.25 = 1.20$ or $7.25x = 1.20$
$x = .1655$ or $x = 16.6\%$ increase

Place Value

$10^1 = 10$	$10^{-1} = 0.1$
$10^2 = 100$	$10^{-2} = 0.01$
$10^3 = 1000$	$10^{-3} = 0.001$
$10^4 = 10000$	$10^{-4} = 0.0001$

Negative and Positive Signs

Adding Rules:

Positive + Positive = Positive: 5 + 4 = 9
Negative + Negative = Negative: (-7) + (-2) = -9
Sum of a negative and a positive number: Use the sign of the larger number and subtract

(-7) + 4 = -3
6 + (-9) = -3
(-3) + 7 = 4
5 + (-3) = 2

Subtracting Rules:

Negative - Positive = Negative:
(-5) - 3 = -5 + (-3) = -8
Positive - Negative = Positive + Positive = Positive:
5 - (-3) = 5 + 3 = 8
Negative - Negative = Negative + Positive = Use the sign of the larger number and subtract *(Change double negatives to a positive)*

(-5) - (-3) = (-5) + 3 = -2
(-3) - (-5) = (-3) + 5 = 2

Multiplying Rules:

Positive x Positive = Positive: 3 x 2 = 6
Negative x Negative = Positive: (-2) x (-8) = 16
Negative x Positive = Negative: (-3) x 4 = -12
Positive x Negative = Negative: 3 x (-4) = -12

Dividing Rules:

Positive ÷ Positive = Positive: 12 ÷ 3 = 4
Negative ÷ Negative = Positive: (-12) ÷ (-3) = 4
Negative ÷ Positive = Negative: (-12) ÷ 3 = -4
Positive ÷ Negative = Negative: 12 ÷ (-3) = -4

Order of Operations

In Mathematics, the order in which mathematical problems are solved is <u>extremely</u> important.

Rules

1. Calculations must be done from left to right.
2. Calculations in brackets (parenthesis) are done first. When you have more than one set of brackets, do the inner brackets first.
3. Exponents (or radicals) must be done next.
4. Multiply and divide in the order the operations occur.
5. Add and subtract in the order the operations occur.

Acronyms to Help You Remember

Please Excuse My Dear Aunt Sally *(Parenthesis, Exponents, Multiply, Divide, Add, Subtract)*

BEDMAS *(Brackets, Exponents, Divide, Multiply, Add, Subtract)*

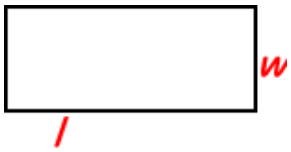
Big Elephants Destroy Mice And Snails *(Brackets, Exponents, Divide, Multiply, Add, Subtract)*

Pink Elephants Destroy Mice And Snails *(Parenthesis, Exponents, Divide, Multiply, Add, Subtract)*

Definitions for Properties of Mathematics

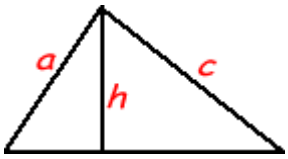
Distributive Property The sum of two numbers times a third number is equal to the sum of each addend times the third number. For example $a \times (b + c) = a \times b + a \times c$	**Associative Property of Addition** When three or more numbers are added, the sum is the same regardless of the grouping of the addends. For example $(a + b) + c = a + (b + c)$
Associative Property of Multiplication When three or more numbers are multiplied, the product is the same regardless of the order of the multiplicands. For example $(a \times b) \times c = a \times (b \times c)$	**Commutative Property of Addition** When two numbers are added, the sum is the same regardless of the order of the addends. For example $a + b = b + a$
Commutative Property of Multiplication When two numbers are multiplied together, the product is the same regardless of the order of the multiplicands. For example $a \times b = b \times a$	**Identity Property of Addition** The sum of any number and zero is the original number. For example $a + 0 = a$.
Identity Property of Multiplication The product of any number and one is that number. For example $a \times 1 = a$.	**Additive Inverse of a Number** The additive inverse of a number, a is -a so that $a + (-a) = 0$.
Multiplicative Inverse of a Number The multiplicative inverse of a number, a is $\frac{1}{a}$ so that $a \times \frac{1}{a} = 1$	**Multiplication Property of Zero** Multiplying any number by 0 yields 0. For example $a \times 0 = 0$.
Addition Property of Zero Adding 0 to any number leaves it unchanged. For example $a + 0 = a$.	**Property of Equality for Addition** Property of Equality for Addition says that if $a = b$, then $a + c = b + c$. If you add the same number to both sides of an equation, the equation is still true
Property of Equality for Multiplication Property of Equality for Multiplication says that if $a = b$, then $a \times c = b \times c$. If you multiply the same number to both sides of an equation, the equation is still true.	

Geometry



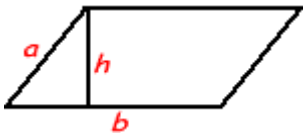
Rectangle

Area = Length X Width A = lw
Perimeter = 2 X Lengths + 2 X Widths P = 2l + 2w



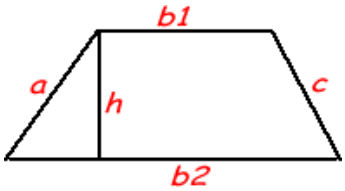
Triangle

Area = 1/2 of the base X the height a = 1/2 bh
Perimeter = a + b + c (add the length of the 3 sides)



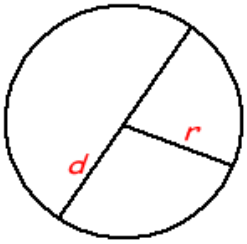
Parallelogram

Area = Base X Height a = bh
Perimeter = 2 X Lengths + 2 X Widths P = 2l + 2w



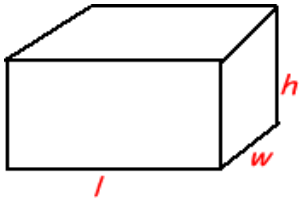
Trapezoid

$A = \left(\frac{b1 + b2}{2}\right)h$
Perimeter = a+b1+b2+c



Circle

The distance around the circle is a circumference. The distance across the circle is the diameter (d). The radius (r) is the distance from the center to a point on the circle. (Pi = 3.14)
$d = 2r$ $c = \pi d = 2\pi r$ $A = \pi r^2$



Rectangular Solid

Volume = Length X Width X Height V = lwh
Surface = 2lw + 2lh + 2wh

Angles

Right Angles: 90 degrees

Straight Angle: 180 degrees

Complementary Angles: Two angles the sum of whose measures is 90 degrees

Supplementary Angles: Two angles the sum of whose measures is 180 degrees

Triangles

Triangles: Sum of the interior angles is 180 degrees

Isosceles Triangle: Two equal sides; two equal angles

Equilateral Triangle: Three equal sides; three equal angles

Right Triangles - Pythagorean Theorem:

$a^2 + b^2 = c^2$, where a and b are the measures of the legs of the triangle and c is the hypotenuse.

Ratio and Proportion

Ratio

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:). Suppose we want to write the ratio of 8 and 12. We can write this as 8:12 or as a fraction 8/12, and we say the ratio is *eight to twelve*.

Examples:

Jeannine has a bag with 3 videocassettes, 4 marbles, 7 books, and 1 orange.

1) What is the ratio of books to marbles?
Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be 7/4.
Two other ways of writing the ratio are 7 to 4, and 7:4.

2) What is the ratio of videocassettes to the total number of items in the bag?
There are 3 videocassettes, and 3 + 4 + 7 + 1 = 15 items total.
The answer can be expressed as 3/15, 3 to 15, or 3:15.

Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

Example:

Are the ratios 3 to 4 and 6:8 equal?
The ratios are equal if 3/4 = 6/8.
These are equal if their cross products are equal; that is, if 3 × 8 = 4 × 6. Since both of these products equal 24, the answer is yes, the ratios are equal.
Remember to be careful! Order matters!
A ratio of 1:7 is not the same as a ratio of 7:1.

Examples:

Are the ratios 7:1 and 4:81 equal? No!
7/1 > 1, but 4/81 < 1, so the ratios can't be equal.

Are 7:14 and 36:72 equal?
Notice that 7/14 and 36/72 are both equal to 1/2, so the two ratios are equal.

Proportion

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.
3/4 = 6/8 is an example of a proportion.
When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Question marks or letters are frequently used in place of the unknown number.

Example:

Solve for n: 1/2 = n/4.
Using cross products we see that 2 × n = 1 × 4 =4, so 2 × n = 4. Dividing both sides by 2, n = 4 ÷ 2 so that n = 2.

Algebra

Basic Properties & Facts

Arithmetic Operations

$ab + ac = a(b+c)$ $\qquad a\left(\frac{b}{c}\right) = \frac{ab}{c}$

$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$ $\qquad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ $\qquad \frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$

$\frac{a-b}{c-d} = \frac{b-a}{d-c}$ $\qquad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{ab+ac}{a} = b+c, \ a \neq 0$ $\qquad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$

Exponent Properties

$a^n a^m = a^{n+m}$ $\qquad \frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

$\left(a^n\right)^m = a^{nm}$ $\qquad a^0 = 1, \quad a \neq 0$

$(ab)^n = a^n b^n$ $\qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$a^{-n} = \frac{1}{a^n}$ $\qquad \frac{1}{a^{-n}} = a^n$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$ $\qquad a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(a^n\right)^{\frac{1}{m}}$

Properties of Radicals

$\sqrt[n]{a} = a^{\frac{1}{n}}$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$ $\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

Properties of Inequalities

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$|a| \geq 0$ $\qquad |-a| = |a|$

$|ab| = |a||b|$ $\qquad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a+b| \leq |a| + |b|$ Triangle Inequality

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$i = \sqrt{-1}$ $\qquad i^2 = -1$ $\qquad \sqrt{-a} = i\sqrt{a}, \quad a \geq 0$

$(a+bi) + (c+di) = a + c + (b+d)i$

$(a+bi) - (c+di) = a - c + (b-d)i$

$(a+bi)(c+di) = ac - bd + (ad+bc)i$

$(a+bi)(a-bi) = a^2 + b^2$

$|a+bi| = \sqrt{a^2+b^2}$ Complex Modulus

$\overline{(a+bi)} = a - bi$ Complex Conjugate

$\overline{(a+bi)}(a+bi) = |a+bi|^2$

Logarithms and Log Properties

Definition

$y = \log_b x$ is equivalent to $x = b^y$

Example

$\log_5 125 = 3$ because $5^3 = 125$

Special Logarithms

$\ln x = \log_e x$ natural log

$\log x = \log_{10} x$ common log

where $e = 2.718281828\mathbf{K}$

Logarithm Properties

$\log_b b = 1 \qquad \log_b 1 = 0$

$\log_b b^x = x \qquad b^{\log_b x} = x$

$\log_b \left(x^r\right) = r \log_b x$

$\log_b (xy) = \log_b x + \log_b y$

$\log_b \left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

The domain of $\log_b x$ is $x > 0$

Factoring and Solving

Factoring Formulas

$x^2 - a^2 = (x+a)(x-a)$

$x^2 + 2ax + a^2 = (x+a)^2$

$x^2 - 2ax + a^2 = (x-a)^2$

$x^2 + (a+b)x + ab = (x+a)(x+b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$

$x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^3$

$x^3 + a^3 = (x+a)\left(x^2 - ax + a^2\right)$

$x^3 - a^3 = (x-a)\left(x^2 + ax + a^2\right)$

$x^{2n} - a^{2n} = \left(x^n - a^n\right)\left(x^n + a^n\right)$

If n is odd then,

$x^n - a^n = (x-a)\left(x^{n-1} + ax^{n-2} + \mathbf{L} + a^{n-1}\right)$

$x^n + a^n$

$= (x+a)\left(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \mathbf{L} + a^{n-1}\right)$

Quadratic Formula

Solve $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ - Two real unequal solns.

If $b^2 - 4ac = 0$ - Repeated real solution.

If $b^2 - 4ac < 0$ - Two complex solutions.

Square Root Property

If $x^2 = p$ then $x = \pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

$|p| = b \quad \Rightarrow \quad p = -b$ or $p = b$

$|p| < b \quad \Rightarrow \quad -b < p < b$

$|p| > b \quad \Rightarrow \quad p < -b$ or $p > b$

Completing the Square

Solve $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of x, square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Functions and Graphs

Constant Function

$y = a$ or $f(x) = a$

Graph is a horizontal line passing through the point $(0, a)$.

Line/Linear Function

$y = mx + b$ or $f(x) = mx + b$

Graph is a line with point $(0, b)$ and slope *m*.

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope *m* and *y*-intercept $(0, b)$ is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope *m* and passing through the point (x_1, y_1) is

$$y = y_1 + m(x - x_1)$$

Parabola/Quadratic Function

$y = a(x-h)^2 + k$ $f(x) = a(x-h)^2 + k$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k).

Parabola/Quadratic Function

$y = ax^2 + bx + c$ $f(x) = ax^2 + bx + c$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Parabola/Quadratic Function

$x = ay^2 + by + c$ $g(y) = ay^2 + by + c$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex at $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$.

Circle

$(x-h)^2 + (y-k)^2 = r^2$

Graph is a circle with radius *r* and center (h, k).

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Graph is an ellipse with center (h, k) with vertices *a* units right/left from the center and vertices *b* units up/down from the center.

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at (h, k), vertices *a* units left/right of center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Hyperbola

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at (h, k), vertices *b* units up/down from the center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
$\left(x^2\right)^3 \neq x^5$	$\left(x^2\right)^3 = x^2x^2x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{\cancel{a}+bx}{\cancel{a}} \neq 1+bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \neq -ax-a$	$-a(x-1) = -ax+a$ Make sure you distribute the "-"!
$(x+a)^2 \neq x^2+a^2$	$(x+a)^2 = (x+a)(x+a) = x^2+2ax+a^2$
$\sqrt{x^2+a^2} \neq x+a$	$5 = \sqrt{25} = \sqrt{3^2+4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3+4 = 7$
$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	See previous error.
$(x+a)^n \neq x^n + a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2\left(x^2+2x+1\right) = 2x^2+4x+2$ $(2x+2)^2 = 4x^2+8x+4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parethesis!
$\sqrt{-x^2+a^2} \neq -\sqrt{x^2+a^2}$	$\sqrt{-x^2+a^2} = \left(-x^2+a^2\right)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$

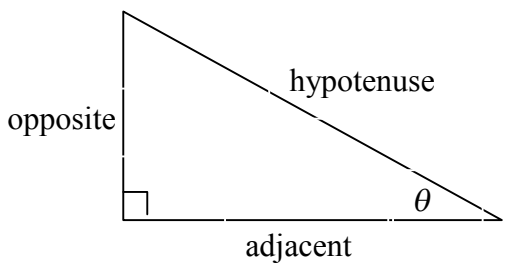
Trigonometry

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



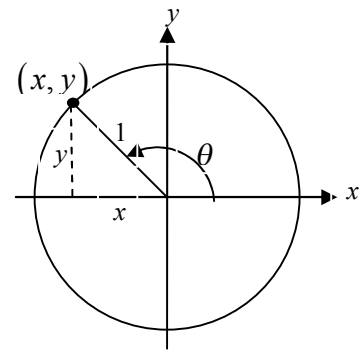
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition

For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\csc\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\cot\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

Range

The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1 \qquad \csc\theta \ge 1$ and $\csc\theta \le -1$

$-1 \le \cos\theta \le 1 \qquad \sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas

$$\sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

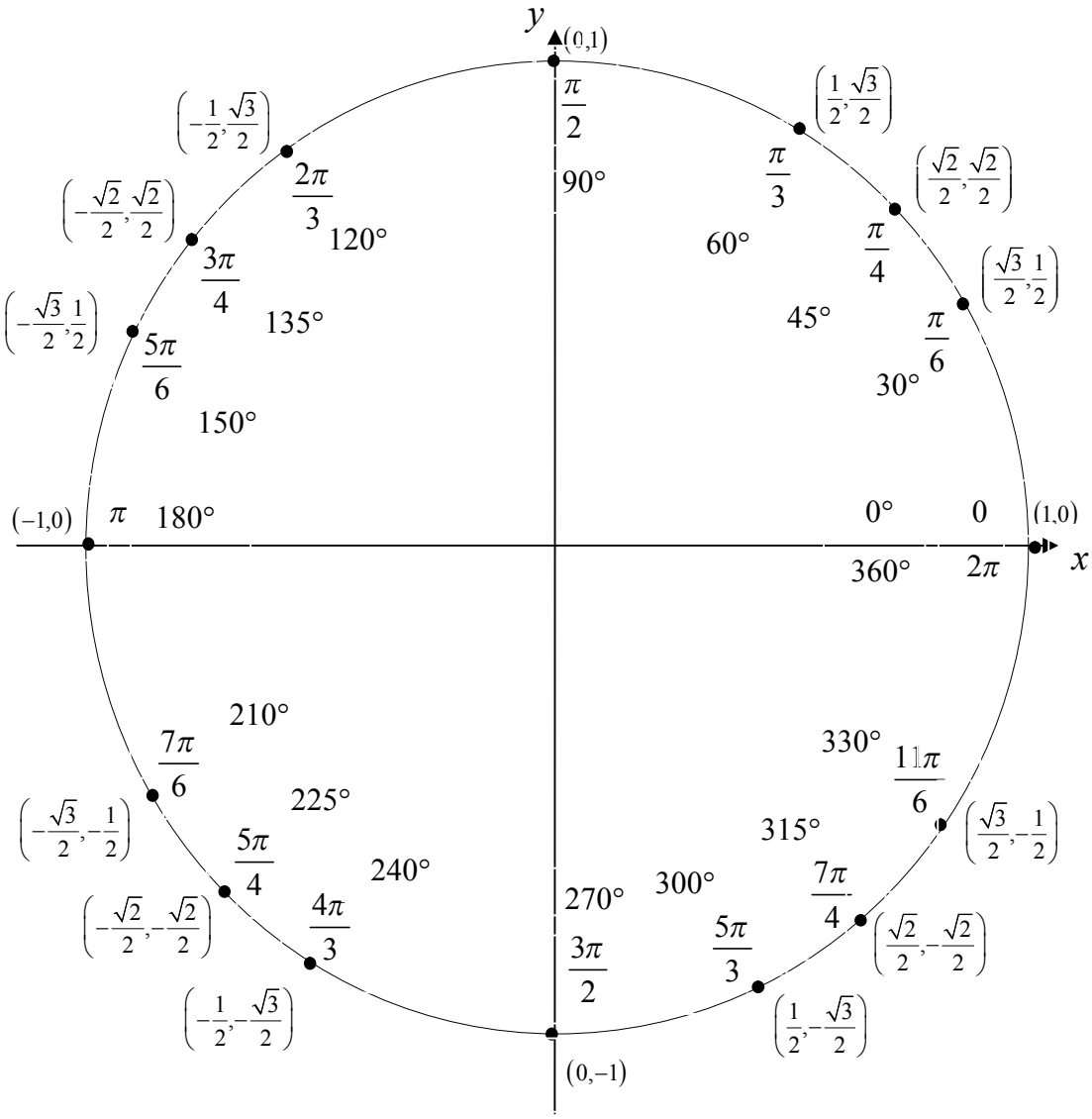
Cofunction Formulas

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}\left(\tan(\theta)\right) = \theta$

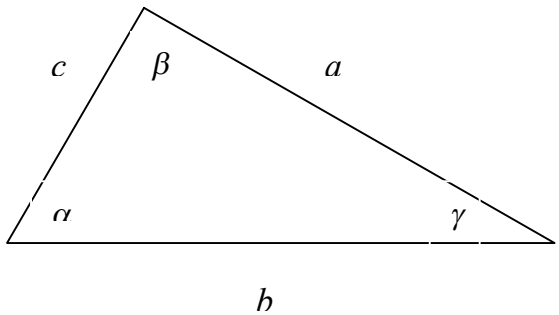
Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$