Problems and Talk Topics for Spring 2022

PROBLEMS AND SOLUTIONS

Edited by **Daniel H. Ullman, Daniel J. Velleman**, and **Douglas B. West**
with the collaboration of Paul Bracken, Ezra A. Brown, Zachary Franco, George Gilbert, László Lipták, Rick Luttmann, Hosam Mahmoud, Frank B. Miles, Lenhard Ng, Rajesh Pereira, Kenneth Stolarsky, Richard Stong, Stan Wagon, Lawrence Washington, and Li Zhou.

Proposed problems should be submitted online at
americanmathematicalmonthly.submittable.com/submit.
Proposed solutions to the problems below should be submitted by February 28, 2022, via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.

PROBLEMS

12272. *Proposed by H. A. ShahAli, Tehran, Iran, and Stan Wagon, Macalester College, St. Paul, MN.*
(a) For which integers n with $n \geq 3$ do there exist distinct positive integers $a_1, \ldots, a_n$ such that $a_i + a_{i+1}$ is a power of 2 for all $i \in \{1, \ldots, n\}$? (Here subscripts are taken modulo n, so that $a_{n+1} = a_1$.)
(b) What is the answer if the word "positive" is removed from part **(a)**?

12273. *Proposed by Hideyuki Ohtsuka, Saitama, Japan.* Let ζ be the Riemann zeta function, defined by $\zeta(s) = \sum_{k=1}^{\infty} 1/k^s$. For $s > 1$, prove the following inequalities:

$$\sum_{\text{prime } p} \frac{1}{p^s - 0.5} < \log \zeta(s), \qquad \sum_{\text{prime } p} \frac{1}{p^s} < \log \frac{\zeta(s)}{\sqrt{\zeta(2s)}}, \qquad \sum_{\text{prime } p} \frac{1}{p^s + 0.5} < \log \frac{\zeta(s)}{\zeta(2s)}.$$

12274. *Proposed by Roberto Tauraso, Università di Roma "Tor Vergata," Rome, Italy.* Evaluate

$$\int_0^1 \frac{\arctan x}{1 + x^2} \left(\ln \left(\frac{2x}{1 - x^2} \right) \right)^2 dx.$$

12275. *Proposed by Yun Zhang, Xi'an, China.* Let x, y, and z be positive real numbers with $x + y + z = 3$. Prove each of the following inequalities.

(a) $x^5 y^5 z^5 (x^4 + y^4 + z^4) \leq 3$.

(b) $x^8 y^8 z^8 (x^5 + y^5 + z^5) \leq 3$.

(c) $x^{11} y^{11} z^{11} (x^6 + y^6 + z^6) \leq 3$.

(d) $x^{16} y^{16} z^{16} (x^7 + y^7 + z^7) \leq 3$.

12276. *Proposed by Joe Santmyer, Las Cruces, NM.* Prove

$$\sum_{n=2}^{\infty} \frac{1}{n+1} \sum_{i=1}^{\lfloor n/2 \rfloor} \frac{1}{2^{i-1}(i-1)!(n-2i)!} = 1.$$

doi.org/10.1080/00029890.2021.1953285

12277. *Proposed by Cristian Chiser, Elena Cuza College, Craiova, Romania.* Let A, B, and C be three pairwise commuting 2-by-2 real matrices. Show that if at least one of the matrices $A - B$, $B - C$, and $C - A$ is invertible, then the matrix

$$A^2 + B^2 + C^2 - AB - AC - BC$$

cannot have rank 1.

12278. *Proposed by Dao Thanh Oai, Thai Binh, Vietnam.* Let ABC be a scalene triangle, and let its external angle bisectors at A, B, and C meet BC, CA, and AB at D, E, and F, respectively. Let l, m, and n be lines through D, E, and F that (internally) trisect angles $\angle ADB$, $\angle BEC$, and $\angle CFA$, respectively, with the angle between l and AD equal to 1/3 of $\angle ADB$, the angle between m and BE equal to 1/3 of $\angle BEC$, and the angle between n and CF equal to 1/3 of $\angle CFA$.
(a) Show that l, m, and n form an equilateral triangle.
(b) The lines l, m, and n each intersect AD, BE, and CF. Of these nine points of intersection, three are the points D, E, and F. Show that the other six lie on a circle.

PROBLEMS

Proposals

To be considered for publication, solutions should be received by March 1, 2022.

2126. *Proposed by M. V. Channakeshava, Bengaluru, India.*

A tangent line to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

meets the x-axis and y-axis at the points A and B, respectively.

Find the minimum value of AB.

2127. *Proposed by Jeff Stuart, Pacific Lutheran University, Tacoma, WA and Roger Horn, Tampa, FL.*

Suppose that $A, B \in M_{n\times n}(\mathbb{C})$ such that $AB = A$ and $BA = B$. Show that

(a) A and B are idempotent and have the same null space.

(b) If $1 \leq \operatorname{rank} A < n$, then there are infinitely many choices of B that satisfy the hypotheses.

(c) $A = B$ if and only if $A - I$ and $B - I$ have the same null space.

2128. *Proposed by George Stoica, Saint John, NB, Canada.*

Let $0 < a < b < 1$ and $\epsilon > 0$ be given. Prove the existence of positive integers m and n such that $(1 - b^m)^n < \epsilon$ and $(1 - a^m)^n > 1 - \epsilon$.

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We invite readers to submit original problems appealing to students and teachers of advanced undergraduate mathematics. Proposals must always be accompanied by a solution and any relevant bibliographical information that will assist the editors and referees. A problem submitted as a Quickie should have an unexpected, succinct solution. Submitted problems should not be under consideration for publication elsewhere.

Proposals and solutions should be written in a style appropriate for this MAGAZINE.

Authors of proposals and solutions should send their contributions using the Magazine's submissions system hosted at `http://mathematicsmagazine.submittable.com`. *More detailed instructions are available there. We encourage submissions in PDF format, ideally accompanied by* LaTeX *source. General inquiries to the editors should be sent to* `mathmagproblems@maa.org`.

2129. *Proposed by Vincent Coll and Daniel Conus, Lehigh University, Bethlehem, PA and Lee Whitt, San Diego, CA.*

Determine whether the following improper integrals are convergent or divergent.

(a) $\displaystyle\int_0^1 \exp\left(\sum_{k=0}^{\infty} x^{2^k}\right)\, dx$

(b) $\displaystyle\int_0^1 \exp\left(\sum_{k=0}^{\infty} x^{3^k}\right)\, dx$

2130. *Proposed by Florin Stanescu, Şerban Cioiculescu School, Găeşti, Romania.*

Given the acute triangle ABC, let D, E, and F be the feet of the altitudes from A, B, and C, respectively. Choose $P, R \in \overleftrightarrow{AB}$, $S, T \in \overleftrightarrow{BC}$, $Q, U \in \overleftrightarrow{AC}$ so that

$$D \in \overleftrightarrow{PQ},\ E \in \overleftrightarrow{RS},\ F \in \overleftrightarrow{TU} \text{ and } \overleftrightarrow{PQ} \parallel \overleftrightarrow{EF},\ \overleftrightarrow{RS} \parallel \overleftrightarrow{DF},\ \overleftrightarrow{TU} \parallel \overleftrightarrow{DE}.$$

Show that

$$\frac{PQ + RS - TU}{AB} + \frac{RS + TU - PQ}{BC} + \frac{TU + PQ - RS}{AC} = 2\sqrt{2}$$

if and only if the circumcenter of $\triangle ABC$ lies on the incircle of $\triangle ABC$.

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This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively *both* by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

Proposed problems should be sent to **Greg Oman**, either by email (preferred) as a pdf, TEX, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

Solutions to the problems in this issue should be sent to **Chip Curtis**, either by email as a pdf, TEX, or Word attachment (preferred) or by mail to the address provided above, no later than March 15, 2022. Sending both pdf and TEX files is ideal.

PROBLEMS

1206. *Proposed by Seán M. Stewart, Bomaderry, NSW, Australia.*

Let $H_n := \sum_{k=1}^{n} \frac{1}{k}$ denote the nth harmonic number, and let F_n denote the nth Fibonacci number, where $F_0 := 0$, $F_1 := 1$, and $F_n := F_{n-1} + F_{n-2}$ for $n \geq 2$. Further, let T_n be the nth triangular number, defined by $T_0 := 0$ and $T_n := n + T_{n-1}$ for $n \geq 1$, and let $\varphi := \frac{1+\sqrt{5}}{2}$ be the golden ratio. Prove the following:

$$\sum_{n=1}^{\infty} \frac{T_n H_n F_n}{2^n} = 52\log(2) + \frac{232}{\sqrt{5}}\log(\varphi) + 73.$$

1207. *Ovidiu Furdui and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.*

Establish the following:

$$\sum_{n=1}^{\infty}(2n-1)\left(\sum_{k=n}^{\infty}\frac{1}{k^2}\right)\left(\sum_{k=n}^{\infty}\frac{1}{k^3}\right) = \zeta(2) + \zeta(3),$$

where for a positive integer k, we have $\zeta(k) = \sum_{n=1}^{\infty} \frac{1}{n^k}$.

doi.org/10.4169/college.math.j.52.4.xxx

1208. *Proposed by Marián Štofka, Slovak University of Technology, Bratislava, Slovakia.*

Prove that

$$\int_0^1 \frac{\ln(1-x)\ln(1+x)}{x}dx = -\frac{5}{8}\zeta(3),$$

where as above, for a positive integer k, we have $\zeta(k) = \sum_{n=1}^{\infty} \frac{1}{n^k}$.

1209. *Proposed by George Stoica, Saint John, New Brunswick, Canada.*

For nonnegative integers i and j, define

$$a_{ij} := \begin{cases} i(i-1)\cdots(i-j+1) & \text{if } 1 \le j \le i, \\ 1 & \text{if } i = 0 \text{ and } j \ge 0, \text{ or } j = 0 \text{ and } i \ge 0, \text{ and} \\ 0 & \text{if } j > i \ge 1. \end{cases}$$

Now let m be a positive integer. Prove that every $m \times m$ submatrix of the infinite matrix $(a_{2i,j})$ with $0 \le j \le m-1$ and $i \ge 0$ has rank m and, in addition, that $\sum_{i=0}^{m}(-1)^i\binom{m}{i}a_{2k+2i,j} = 0$ for $0 \le j \le m-1$ and any nonnegative integer k.

1210. *Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO.*

Let R be a commutative ring with identity, and let I and J be ideals of R. Recall that the *sum* of I and J is the ideal defined by $I + J := \{i + j : i \in I,\ j \in J\}$. Prove or disprove: there exists a countable commutative integral domain D with identity and a collection $\mathcal{S}$ of $2^{\aleph_0}$ ideals of D such that for all $I \neq J$ in $\mathcal{S}$, we have $I + J \notin \mathcal{S}$.

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PROBLEMS

12279. *Proposed by Brad Isaacson, Brooklyn, NY.* Let $S(m, k)$ denote the number of partitions of a set with m elements into k nonempty blocks. (These are the *Stirling numbers of the second kind*.) Let j and n be positive integers of opposite parity with $j < n$. Prove

$$\sum_{r=j}^{n} \frac{(-1)^r (r-1)! \binom{r}{j} S(n, r)}{2^r} = 0.$$

12280. *Proposed by Nguyen Duc Toan, Da Nang, Vietnam.* Let ABC be an acute scalene triangle with circumcenter O and orthocenter H. Let M and R be the midpoints of segments BC and OH, respectively, let S be the reflection across BC of the circumcenter of triangle BOC, and let T be the second point of intersection of the circumcircle of triangle BHC and line OH. Prove that M, R, S, and T are concyclic.

12281. *Proposed by Paolo Perfetti, Università di Roma "Tor Vergata," Rome, Italy.* Evaluate

$$\int_0^\infty \left(\frac{\cosh x}{\sinh^2 x} - \frac{1}{x^2} \right) (\ln x)^2 \, dx.$$

12282. *Proposed by George Stoica, Saint John, NB, Canada.* Prove that the multiplicative group generated by $\{\lfloor \sqrt{2}n \rfloor / n : n \in \mathbb{Z}^+\}$ is the group of positive rational numbers.

12283. *Proposed by Yongge Tian, Shanghai Business School, Shanghai, China.* Let A and B be two n-by-n matrices that are orthogonal projections, that is, $A^2 = A = A^*$ and $B^2 = B = B^*$. Let $\sqrt{A+B}$ denote the positive semidefinite square root of $A + B$. Prove

$$\begin{aligned} \operatorname{trace}(A+B) - (2-\sqrt{2})\operatorname{rank}(AB) &\le \operatorname{trace}\sqrt{A+B} \\ &\le (\sqrt{2}-1)\operatorname{trace}(A+B) + (2-\sqrt{2})\operatorname{rank}(A+B), \end{aligned}$$

and show that equality holds simultaneously if and only if $AB = BA$.

doi.org/10.1080/00029890.2021.1964307

12284. *Proposed by Zachary Franco, Houston, TX.* Let ABC be a triangle with circumcenter O, incenter I, orthocenter H, sides of integer length, and perimeter 2021. Suppose that the perpendicular bisector of OH contains A and I. Find the length of BC.

12285. *Proposed by Atul Dixit, Indian Institute of Technology, Gandhinagar, India.* Prove

$$\sum_{m=1}^{\infty}\int_0^{\infty}\frac{t\cos t}{t^2+m^2u^2}\,dt=\int_0^{\infty}\left(-\frac{\pi}{2u}\cos t+\sum_{m=1}^{\infty}\frac{t\cos t}{t^2+m^2u^2}\right)dt$$

for $u>0$.

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12286. *Proposed by Ira Gessel, Brandeis University, Waltham, MA.* Let p be a prime number, and let m be a positive integer not divisible by p. Show that the coefficients of $(1+x+\cdots+x^{m-1})^{p-1}$ that are not divisible by p are alternately 1 and -1 modulo p. For example, $(1+x+x^2+x^3)^4 \equiv 1-x+x^4-x^6+x^8-x^{11}+x^{12} \pmod 5$.

12287. *Proposed by Ovidiu Furdui and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.* Prove

$$\sum_{n=1}^{\infty}\left(n\left(\sum_{k=n}^{\infty}\frac{1}{k^2}\right)^2-\frac{1}{n}\right)=\frac{3}{2}-\frac{1}{2}\zeta(2)+\frac{3}{2}\zeta(3),$$

where ζ is the Riemann zeta function, defined by $\zeta(s)=\sum_{k=1}^{\infty}1/k^s$.

12288. *Proposed by Seán Stewart, Bomaderry, Australia.* Prove

$$\int_0^{\infty}\left(1-x^2\sin^2\left(\frac{1}{x}\right)\right)^2dx=\frac{\pi}{5}.$$

12289. *Proposed by George E. Andrews, Pennsylvania State University, University Park, PA, and Mircea Merca, University of Craiova, Craiova, Romania.* Prove

$$\sum_{n=0}^{\infty}2\cos\left(\frac{(2n+1)\pi}{3}\right)q^{n(n+1)/2}=\prod_{n=1}^{\infty}(1-q^n)(1-q^{6n-1})(1-q^{6n-5}),$$

when $|q|<1$.

12290. *Proposed by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.* Find all analytic functions $f:\mathbb{C}\to\mathbb{C}$ that satisfy

$$|f(x+iy)|^2=|f(x)|^2+|f(iy)|^2$$

for all real numbers x and y.

http://dx.doi.org/doi.org/10.1080/00029890.2021.1982223

12291. *Proposed by Leonard Giugiuc, Drobeta Turnu Severin, Romania, and Petru Braica, Satu Mare, Romania.* The *Nagel point* of a triangle is the point common to the three segments that join a vertex of the triangle to the point at which an excircle touches the opposite side. Let ABC be a triangle with incenter I and Nagel point J. Prove that AJ is perpendicular to the line through the orthocenters of triangles IAB and IAC.

12292. *Proposed by Nikolai Osipov, Siberian Federal University, Krasnoyarsk, Russia.* Let p be a prime number, and let $r = 1/(2\cos(4\pi/7))$. Evaluate $\lfloor r^{p+2} \rfloor$ modulo p.

Talk Topics for 2021-2022 School Year

Below are some possible topics. These suggestions include areas of Math, Applied Math, Math History, and Math Education. I hope these include something for each taste. (I can also make other suggestions.)

1. With COVID-19 epidemic, the interest in mathematical models of diseases is on the rise. They span statistical, discrete, and continuous models. For example, discrete or continuous SIR or SEIR models. Any overview talk of such models or specific development of models for COVID-19 will make an interesting talk.
2. Fibonacci Day is Nov 23 (1123). A talk on properties and derivation of Fibonacci sequence formula will be very informative. There are many sources for this and can be found easily.
3. Almost Perfect Numbers - For a natural number n, call sum of its proper divisors s(n). A natural number is perfect if it is sum of its proper divisors: s(n)=n. For example, 6 is a perfect number since 1+2+3=6. A natural number is called almost perfect (also called perfect-minus-one) if sum of its proper divisors is one less than the number: s(n)=n-1. For example, 8 is an almost perfect number since 1+2+4=8-1. The only known almost perfect numbers are of powers of 2. The only known odd almost perfect number is $2^0 = 1$. The research project is to quantify, as much as possible, almost perfect numbers. This investigation can include both theoretical and computational work. References: http://www.jstor.org/stable/2689036, http://www.jstor.org/stable/2303162 and http://www.ams.org/journals/mcom/1981-36-154/S0025-5718-1981-0606516-3/.
4. Geometric Solutions of Cubic Equations - The 11th century Persian poet, mathematician, and astronomer Omar Khayyam devised a geometrical method to also solve the cubic equation $ax^3 + bx^2 + cx + d = 0$. There are many references for this problem.
5. Italian mathematician Margherita Piazzolla Beloch showed that a fold operation can be used to solve arbitrary cubic equations and so can be tied to the above. Now, Beloch's fold is also one of the "axioms" of origami. References: Beloch, M.P. (1936) Sul metodo del ripiegamento della carta per la risoluzione dei problemi geometrici. Periodico di Mathematiche Ser. 4, 16: 104–108 and Hull, T.C. (2011) Solving cubics with creases: The work of Beloch and Lill, Amer. Math. Monthly,118, 307–315.
6. Present the paper *Existence of a Solution for Beloch's Fold* is by Jorge C. Lucero, Mathematics Magazine, vol. 92 (2019), no. 1, 24-31, is available at https://maa.tandfonline.com/doi/full/10.1080/0025570X.2019.1526591.
7. Proof of irrationality of number pi, for the Pi Day (Mar 14, 3/14). There are many sources for this and can be found easily.

8. Elementary Methods for Solving Calculus Problems - Many calculus problems can be solved using algebra and geometry, without using the mathematical tools developed in calculus. The Armenian astrophysicist Mamikon A. Mnatsakanian went to Caltech after the fall of Soviet Union and returned to his first love of developing geometric and visual methods for solving calculus problems. There he worked with the famous American mathematician Tom M. Apostol at "Project MATHEMATICS!". Any of their papers include a wealth of insight and are great for presentations at any level. The first paper was titled "A Visual Approach to Calculus Problems" which you can find below. References: http://calteches.library.caltech.edu/4007/1/Calculus.pdf, http://www.its.caltech.edu/~mamikon/calculus.html and http://www.projectmathematics.com/.
9. Short biographical talks on female mathematicians, for example, Margherita Piazzolla Beloch, Elizabeth Smith, etc. See https://en.wikipedia.org/wiki/List_of_women_in_mathematics.
10. Present the paper *An Alternative to Integration by Partial Fractions Technique*. This is the title of a short paper by Yusuf Gurtas published in The College Mathematics Journal, vol. 50 (2019), no. 2, 140-142, and available at https://www.tandfonline.com/doi/full/10.1080/07468342.2019.1561125.
11. Proof of Convergence of Fourier Series. Read, understand, and rewrite the proof in your own words. Reference: David Powers, BVP's and PDE's, 6th edition, ISBN 978-0-12-374719-8.
12. Presentation of the calculus based proof of $\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\cdots=\frac{\pi^2}{6}$. You learn in Calculus II that this infinite series in convergent (p-series with $p=2>1$). In Boundary Value Problems, you use Fourier series to find its value. But this can be done elegantly using calculus and is included in the book *Proofs from THE BOOK* by Martin Aigner and Günte Ziegler, available in the library. However, there are other proofs. The paper http://math.cmu.edu/~bwsulliv/basel-problem.pdf lists several solutions with references.
13. Present the paper *A Simple Proof of Descartes' Rule of Sign*. This is the title of a short paper by Xiaoshen Wang published in Amer. Math. Monthly, vol. 111 (2004), no. 4, 525-526, and available at https://www.tandfonline.com/doi/abs/10.1080/00029890.2004.11920108.
14. Present the paper *Finding Real Roots of Polynomials Using Sturm Sequences*. This article compares Descartes' Rule of Signs, the Budan-Fourier Theorem, and versions of Sturm's Method in contrast with the approximate root count gleaned from graphing utilities. This articles is published at PRIMUS, (30)1:36-49, 2020, and also available at https://www.tandfonline.com/doi/full/10.1080/10511970.2018.1501626.
15. Present the paper *Inflating the Cube Without Stretching*. This is the title of a short paper by Igor Pak published in Amer. Math. Monthly, vol. 115 (2008), no. 5, 443-445, and available at http://www.math.ucla.edu/~pak/papers/milka2.pdf. In this article the author describes a way to deform a cube that distances between points are maintained while the volume is increased. The beauty of this article is that one can actually construct the deformed cube!

16. Present the paper Absent-Minded Passengers published in American Mathematical Monthly, 126:10,867-875, and available at https://www.tandfonline.com/doi/full/10.1080/00029890.2019.1656024. Here is the abstract of the paper. Passengers board a fully booked airplane in order. The first passenger picks one of the seats at random. Each subsequent passenger takes his or her assigned seat if available, otherwise takes one of the remaining seats at random. It is well known that the last passenger obtains her own seat with probability 1/2. We study the distribution of the number of incorrectly seated passengers, and we also discuss the case of several absent-minded passengers.

17. Address the conjectures in the paper https://maa.tandfonline.com/doi/pdf/10.1080/0025570X.2020.1704613 regarding certain primes. One such prime is 29 for which $2(29) + 9 = 67$ is also a prime and again $6(67) + 7 = 409$ is another prime.

18. The Arithmetic Mean – Geometric Mean (AM-GM) Inequality has many proofs and applications. A new proof is at https://maa.tandfonline.com/doi/pdf/10.1080/07468342.2020.1697605. An overview of this topic and presentation of interesting proofs will make a nice talk.